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AND

## DISCUSSIONS

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

# ANALYSIS OF RIGID FRAMES BY SUPERPOSITION

By David M. Wilson,[1] M. Am. Soc. C. E.

## Synopsis

The purpose of this paper is to demonstrate the usefulness of the principle of superposition in the analysis of rigid frames by the slope-deflection and moment distribution methods. A number of examples are presented to show how the principle may be used to advantage in attacking a variety of problems. The principle of superposition is usually stated in textbooks on structural analysis; but, in the writer's opinion, its application is not emphasized to the extent that its importance warrants. In the study of continuous structures, in particular, a thorough working knowledge of the principle is indispensable.

It will be noted that no new method of analysis is proposed. Instead, a method of procedure is outlined which coordinates the principle of superposition with existing methods and, at the same time, places emphasis where it properly belongs.

## Preliminary Considerations

The scope of the paper is limited to the analysis of frames in which applied loads cause linear displacements of the joints, in addition to angular displacements (rotations), and consequent angular displacements of the members. It is assumed that each member of any frame considered has constant section throughout its length. However, the method of procedure which is presented is general, since it is based upon fundamental principles, and can be applied with little additional labor to the analysis of frames having members of varying section after fixed-end moments, carry-over factors, and stiffness values have been evaluated.

*Notation.*—The following letter symbols, used in this paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials,[2] prepared by a Committee of the American

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by July 1, 1944.

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[2] ASA—Z10a—1932.

Standards Association, with Society representation, and approved by the Association in 1932:

$C$ = a moment symbol commonly used in slope-deflection analyses; for example, $C_{AB}$ = a fixed-end moment at end A of any member AB, caused by transverse loads on the member (see also $M$);

$E$ = modulus of elasticity;

$K$ = a ratio of moment of inertia of a section to the length of the member = $\frac{I}{L}$;

$L$ = length of any member;

$M$ = bending moment; for example, $M_{AB}$ = resisting moment of end A of any member AB; $M'_{AB}$ = fixed-end moment, caused by the angular displacement of the member (see also $C$);

$R$ = angular displacement, in radians, of the straight line connecting the ends of the member = $\frac{\Delta}{L}$ (referred to as the angular displacement of the member);

$\Delta$ = relative linear displacement of one end of any member with respect to its other end, measured perpendicularly to the initial position of the member;

$\theta$ = angular distance or rotation; for example, $\theta_A$ = the rotation angle at joint A of any frame, or of end A of any member framing into joint A.

*Convention of Signs.*—The signs of the quantities used in the problems of this paper are determined by the following conventional rules:

The rotation angle, $\theta$, is positive when it occurs in a clockwise direction and negative when it occurs in a counterclockwise direction.

The linear displacement, $\Delta$, and the angular displacement, $R$, are both positive when the straight line connecting the ends of the member under consideration rotates in a clockwise direction and negative when this line rotates in a counterclockwise direction.

The resisting moment, $M_{AB}$, and the fixed-end moments, $M'_{AB}$ and $C_{AB}$, are positive when they act in a clockwise direction and negative when they act in a counterclockwise direction.

*Lengths of Members.*—In the solutions of subsequent problems, the lengths of members are taken between axial intersections of members. Therefore, the moments obtained represent the moments at these intersections, not the moments at the faces of supporting members. The latter may be computed without difficulty, however, after the moments and shears at intersections have been determined.

*Fixed-End Moments Induced by Angular Displacement, R.*—The resisting moments at ends A and B, respectively, of any member AB are defined by the slope-deflection equations:

$$M_{AB} = 2\,E\,K\,(2\,\theta_A + \theta_B - 3\,R) + C_{AB} \quad \ldots\ldots\ldots\ldots (1a)$$

and

$$M_{BA} = 2\,E\,K\,(2\,\theta_B + \theta_A - 3\,R) + C_{BA} \quad \ldots\ldots\ldots\ldots (1b)$$

These equations may be expressed in the form:

$$M_{AB} = 2\,E\,K\,(2\,\theta_A + \theta_B) + (C_{AB} - 6\,E\,K\,R) \quad \ldots\ldots\ldots (2a)$$

and

$$M_{BA} = 2\,E\,K\,(2\,\theta_B + \theta_A) + (C_{BA} - 6\,E\,K\,R) \quad \ldots\ldots\ldots (2b)$$

Now, if both ends of the member are fixed against rotation so that $\theta_A = \theta_B = 0$, the fixed-end moments at A and B are obviously equal to $(C_{AB} - 6\,E\,K\,R)$ and $(C_{BA} - 6\,E\,K\,R)$, respectively. Furthermore, if the member carries no transverse loads so that $C_{AB} = C_{BA} = 0$, the fixed-end moments at A and B are each equal to $(-6\,E\,K\,R)$. Therefore,

$$M'_{AB} = M'_{BA} = -6\,E\,K\,R = -6\,E\,K\,\frac{\Delta}{L} \quad \ldots\ldots\ldots\ldots (3)$$

If one end (say, B) of the member is free to rotate and A is fixed against rotation so that $M_{BA} = 0$ and $\theta_A = 0$, it may be readily shown that the fixed-end moment at A, if the member carries no transverse loads, is equal to $(-3\,E\,K\,R)$. Under this condition, the rotational stiffness at A of the member is equal to three fourths the rotational stiffness at A when B is fixed against rotation.

*Outline of Method of Procedure.*—In the problems herein considered, the key to analysis in each case consists in the consideration of all possible conditions of deformation of the given frame. In accordance with the principle of superposition the resultant effects on the frame will be equal to the sum of the component effects produced by these separate deformations. The following outline of method of procedure presents, in logical order, the steps required for the solution of any rigid frame problem:

Step 1. Sketches showing all possible conditions of deformation of the frame are made. If transverse loads act on any member of the frame, these sketches will include one of the undeflected frame with any transverse loads that may be acting.

Step 2. Fixed-end moments set up by the transverse loads and unknown angular displacements of the members, before rotations of the joints are permitted, are determined and recorded on the appropriate sketches of step 1. These moments will be numerical values for the condition of transverse loads on members of the frame, and functions of the unknown angular displacements (or unknown joint displacements) for all other conditions of deformation (see Eq. 3).

Step 3. The joints are balanced for each condition of deformation in step 1 by either the slope-deflection or moment distribution method, and the final moments are shown on the deformation sketches.

Step 4. Equilibrium equations are written for the frame with the given loads; and, after proper substitutions, the unknown displacements are evaluated. These substitutions are determined from a consideration of appropriate free bodies on which the moments found in step 3 and all forces are shown acting. The moment at the end of any member in a free body must equal the sum of the moments obtained at the end of that member for the several de-

formation conditions. It will include a numerical value, if transverse loads act on any member, and additional values that are functions of the unknown displacements.

Step 5. With all displacements known, the moments in step 3 which are functions of these displacements are evaluated, and the final numerical values of the moments at the ends of all members are determined by summation.

It is important to observe that simultaneous equations obtained in the foregoing procedure will usually be the type which may conveniently be solved by successive approximations (method of iteration). This is of particular significance in problems involving more than three unknowns.

## Rectangular Frames

Most frames in practice are rectangular in shape. In general, they must be designed to resist both vertical and horizontal loads. The following examples will illustrate the analysis of rectangular frames with different types of loading.

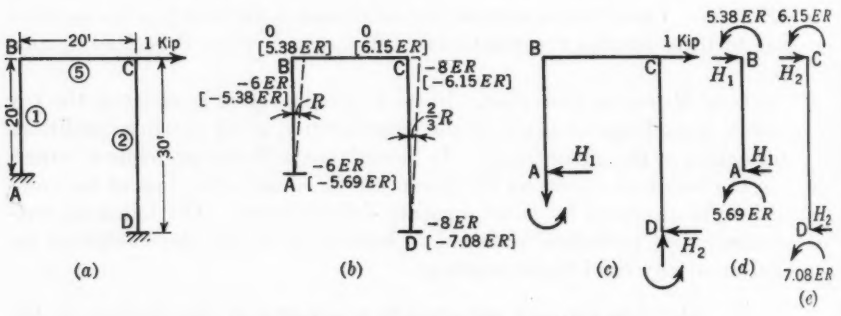


Fig. 1.—Bent with Legs of Unequal Length

*Example 1. Bent with Legs of Unequal Length.*—Fig. 1(*a*) represents a bent with legs of unequal length, subjected to a horizontal force of 1,000 lb at C. The relative *K*-values of all members are shown in circles. It is desired to compute the values of the moments at the ends of all members of the bent.

Step 1. The only possible condition of deformation of the bent is shown in Fig. 1(*b*). It is obvious that $R_{CD} = \frac{2}{3} R_{AB}$.

Step 2. The fixed-end moments at the ends of all members are determined by use of Eq. 3 as functions of $E\,R$ and are recorded in Fig. 1(*b*).

Step 3. In this problem, the joints in Fig. 1(*b*) are balanced by the slope-deflection method.

The moments at the ends of all members are defined by the slope-deflection equations as follows:

$$\left.\begin{aligned} M_{AB} &= 2\,E\,\theta_B - 6\,E\,R \\ M_{BA} &= 4\,E\,\theta_B - 6\,E\,R \\ M_{BC} &= 20\,E\,\theta_B + 10\,E\,\theta_C \\ M_{CB} &= 20\,E\,\theta_C + 10\,E\,\theta_B \\ M_{CD} &= 8\,E\,\theta_C - 8\,E\,R \\ M_{DC} &= 4\,E\,\theta_C - 8\,E\,R \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

At joint B: $M_{BA} + M_{BC} = 0$. Therefore,

$$24\,E\,\theta_B + 10\,E\,\theta_C = 6\,E\,R \quad \ldots\ldots (5a)$$

Also, at joint C: $M_{CB} + M_{CD} = 0$; and

$$10\,E\,\theta_B + 28\,E\,\theta_C = 8\,E\,R \quad \ldots\ldots (5b)$$

Solving Eqs. 5 simultaneously,

$$E\,\theta_B = 0.154\,E\,R \quad \ldots\ldots (6a)$$

and

$$E\,\theta_C = 0.231\,E\,R \quad \ldots\ldots (6b)$$

Substituting Eqs. 6 in Eqs. 4, the following values of the moments are obtained in terms of $E\,R$: $M_{AB} = -5.69\,E\,R$; $M_{BA} = -5.38\,E\,R$; $M_{BC} = 5.38\,E\,R$; $M_{CB} = 6.15\,E\,R$; $M_{CD} = -6.15\,E\,R$; and $M_{DC} = -7.08\,E\,R$. These moments are shown in brackets in Fig. 1(*b*).

Step 4. To determine the value of $E\,R$, an equilibrium equation for the bent must be written.

In Fig. 1(*c*), from $\Sigma F_x = 0$,

$$H_1 + H_2 = 1{,}000 \quad \ldots\ldots (7)$$

In Fig. 1(*d*), from $\Sigma M = 0$,

$$H_1 = \frac{11.07\,E\,R}{20} \quad \ldots\ldots (8a)$$

Similarly, in Fig. 1(*e*),

$$H_2 = \frac{13.23\,E\,R}{30} \quad \ldots\ldots (8b)$$

Substituting Eqs. 8 in Eq. 7, the value of $E\,R$ may be found: $\frac{11.07\,E\,R}{20} + \frac{13.23\,E\,R}{30} = 1{,}000$. Solving, $E\,R = 1{,}005$.

Step 5. The final numerical values of the moments in foot-pounds are: $M_{AB} = -(5.69)(1{,}005) = -5{,}720$; $M_{BA} = -(5.38)(1{,}005) = -5{,}410$; $M_{BC} = (5.38)(1{,}005) = 5{,}410$; $M_{CB} = (6.15)(1{,}005) = 6{,}180$; $M_{CD} = -(6.15) \times (1{,}005) = -6{,}180$; and $M_{DC} = -(7.08)(1{,}005) = -7{,}110$.

*Example 2. Unsymmetrical Three-Column Bent.*—Fig. 2(*a*) represents an unsymmetrical three-column bent with a vertical load of 1,000 lb acting at the center of BC.

Step 1. Two possible conditions of deformation exist—one for the transverse load on BC, and the other for lateral deflection (sidesway) of the frame. These are shown in Figs. 2(*b*) and 2(*c*).

Step 2. Fixed-end moments are computed for the deformation conditions of Figs. 2(*b*) and 2(*c*) and recorded as shown.

Step 3. In this problem, and in subsequent problems, the joints are balanced by the moment distribution method. Fig. 3 shows the distributions of the fixed-end moments shown in Figs. 2(*b*) and 2(*c*). The final moments obtained from these distributions are shown in brackets in Figs. 2(*b*) and 2(*c*).

Step 4. To determine the value of $E\,R$, an equilibrium equation for the bent must be written.

In Fig. 2(*d*), from $\Sigma F_x = 0$,

$$H_1 + H_2 + H_3 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

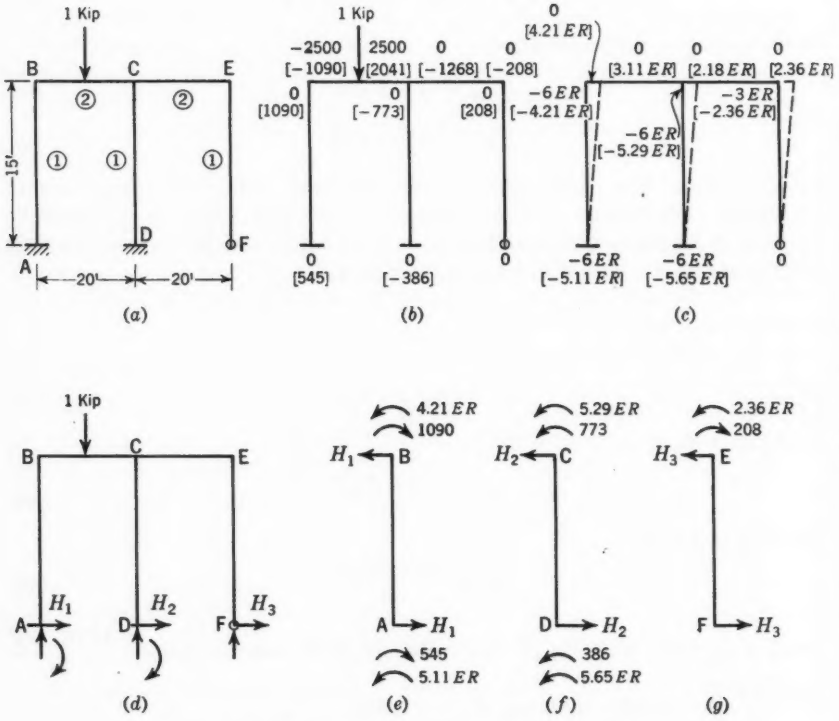


FIG. 2.—UNSYMMETRICAL THREE-COLUMN BENT

In Fig. 2(*e*), from $\Sigma M = 0$, $15\,H_1 + 4.21\,E\,R + 5.11\,E\,R - 1{,}090 - 545 = 0$; and

$$H_1 = \frac{1{,}635 - 9.32\,E\,R}{15} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10a)$$

Similarly, in Fig. 2(*f*),

$$H_2 = \frac{-\,1{,}159 - 10.94\,E\,R}{15} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10b)$$

and, in Fig. 2(*g*),

$$H_3 = \frac{208 - 2.36\,E\,R}{15} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10c)$$

Substituting Eqs. 10 in Eq. 9, the value of $E\,R$ may be found; thus, $684 - 22.62\,E\,R = 0$ and $E\,R = 30.2$.

Step 5. The numerical values of the moments caused by lateral deflection are determined by substituting the value of $E\,R$ in the moment terms shown

FIG. 3.—MOMENT DISTRIBUTION FOR EXAMPLE 2

in brackets in Fig. 2(*c*). By summation of the moments thus obtained and those shown in brackets in Fig. 2(*b*), the following final moments in foot-pounds are computed: $M_{AB} = 390$; $M_{BA} = 963$; $M_{BC} = -963$; $M_{CB} = 2{,}135$; $M_{CD} = -933$; $M_{DC} = -557$; $M_{CE} = -1{,}202$; $M_{EC} = -137$; and $M_{EF} = 137$.

*Example 3. Two-Story Bent with Horizontal Loads.*—Fig. 4(*a*) represents a two-story bent with horizontal loads applied at joints B and C.

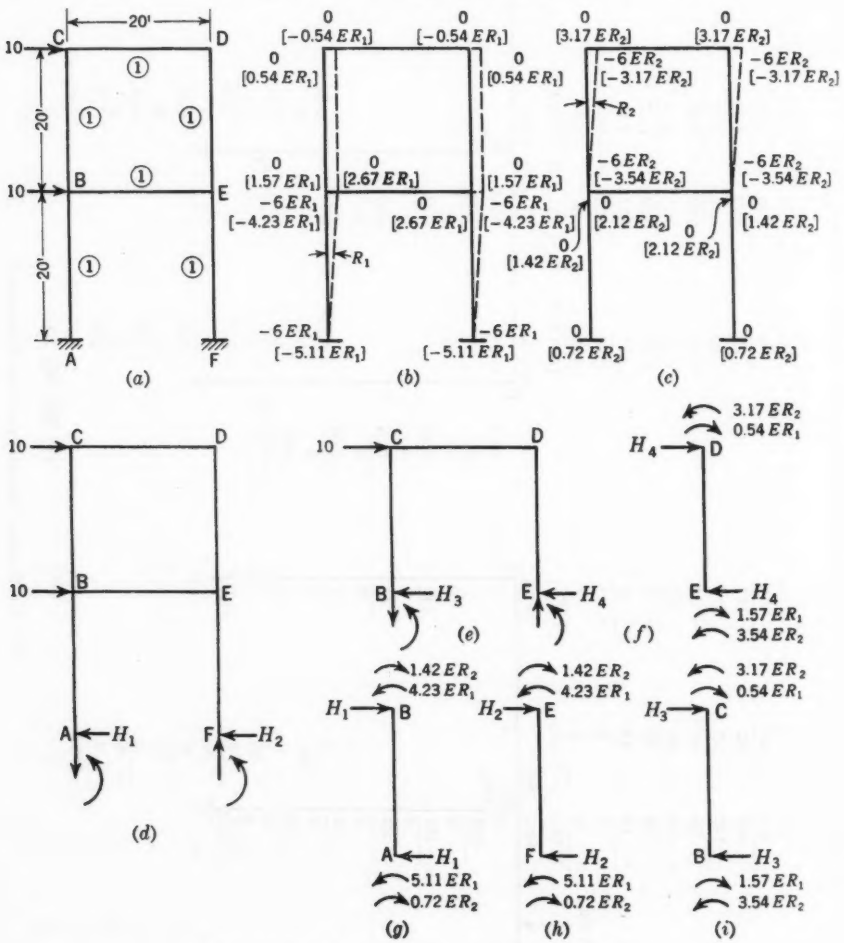


FIG. 4.—TWO-STORY BENT WITH HORIZONTAL LOADS

Step 1. Two possible conditions exist—one for deformation of the first story, and the other for deformation of the second story of the frame. These are shown in Figs. 4(*b*) and 4(*c*).

Step 2. Fixed-end moments are computed as functions of $E\,R_1$ and $E\,R_2$ for the deformation conditions of Figs. 4(*b*) and 4(*c*) and recorded as shown.

Step 3. The joints in Fig. 4(*b*) and 4(*c*) are balanced by the moment distribution method, and the final moments, in terms of $E\,R_1$ and $E\,R_2$, respectively, are shown in brackets. The actual work of distribution is not shown in this or in subsequent examples, inasmuch as routine computations only are involved.

Step 4. To determine the values of $E\,R_1$ and $E\,R_2$, two equilibrium equations for the bent must be written. In Fig. 4(*d*), from $\Sigma F_x = 0$,

$$H_1 + H_2 = 20 \quad \ldots\ldots (11a)$$

and, in Fig. 4(*e*), from $\Sigma F_x = 0$,

$$H_3 + H_4 = 10 \quad \ldots\ldots (11b)$$

In Figs. 4(*g*) and 4(*h*), from $\Sigma M = 0$,

$$H_1 = H_2 = \frac{9.34\,E\,R_1 - 2.14\,E\,R_2}{20} \quad \ldots\ldots (12a)$$

Similarly, in Figs. 4(*i*) and 4(*f*),

$$H_3 = H_4 = \frac{-2.11\,E\,R_1 + 6.71\,E\,R_2}{20} \quad \ldots\ldots (12b)$$

Substituting Eqs. 12 in Eqs. 11,

$$18.68\,E\,R_1 - 4.28\,E\,R_2 = 400 \quad \ldots\ldots (13a)$$

and

$$-4.22\,E\,R_1 + 13.42\,E\,R_2 = 200 \quad \ldots\ldots (13b)$$

Solving Eqs. 13 simultaneously, $E\,R_1 = 26.7$; and $E\,R_2 = 23.4$.

Step 5. The numerical values of the moments caused by each deformation are determined by substituting the values of $E\,R_1$ and $E\,R_2$ in the moment terms shown in brackets in Figs. 4(*b*) and 4(*c*), and the final moments at the ends of all members in foot-pounds are computed by summation of these values: $M_{AB} = -120$; $M_{BA} = -80$; $M_{BE} = 120$; $M_{BC} = -40$; and $M_{CB} = -60$.

*Example 4. Complex Three-Column Bent.*—Fig. 5(*a*) represents a complex three-column bent (5*a*)[3] with a horizontal load of 500 lb applied at joint B.

Step 1. Two possible conditions of deformation exist—one for the part ABCD of the bent, and the other for primary deformation of part EDFG which in turn causes secondary deformation of part ABCD. These are shown in Figs. 5(*b*) and 5(*c*), respectively.

Step 2. Fixed-end moments are computed for the deformation conditions of Figs. 5(*b*) and 5(*c*) and recorded as shown. In this problem, the fixed-end moments are expressed in terms of the unknown linear displacements $\Delta$ instead of the angular displacements $R$ as in previous problems. The method of solution, however, is not changed in any way.

Step 3. The joints in Figs. 5(*b*) and 5(*c*) are balanced by the moment distribution method, and the final moments, in terms of $E\,\Delta_1$ and $E\,\Delta_2$, respectively, are shown in brackets.

Step 4. To determine the values of $E\,\Delta_1$ and $E\,\Delta_2$, two equilibrium equations must be written for the bent. In Fig. 5(*d*), from $\Sigma F_x = 0$,

$$H_1 + H_2 = 500 \quad \ldots\ldots (14a)$$

[3] Numerals in parentheses, thus: (5*a*), refer to corresponding items in the Bibliography (see Appendix).

and in Fig. 5(*e*), from $\Sigma F_x = 0$,

$$H_1 + H_3 + H_4 = 500 \quad \text{.....(14b)}$$

In Fig. 5(*f*), from $\Sigma M = 0$,

$$H_1 = 0.359\ E\ \Delta_1 + 0.269\ E\ \Delta_2 \quad \text{.....(15a)}$$

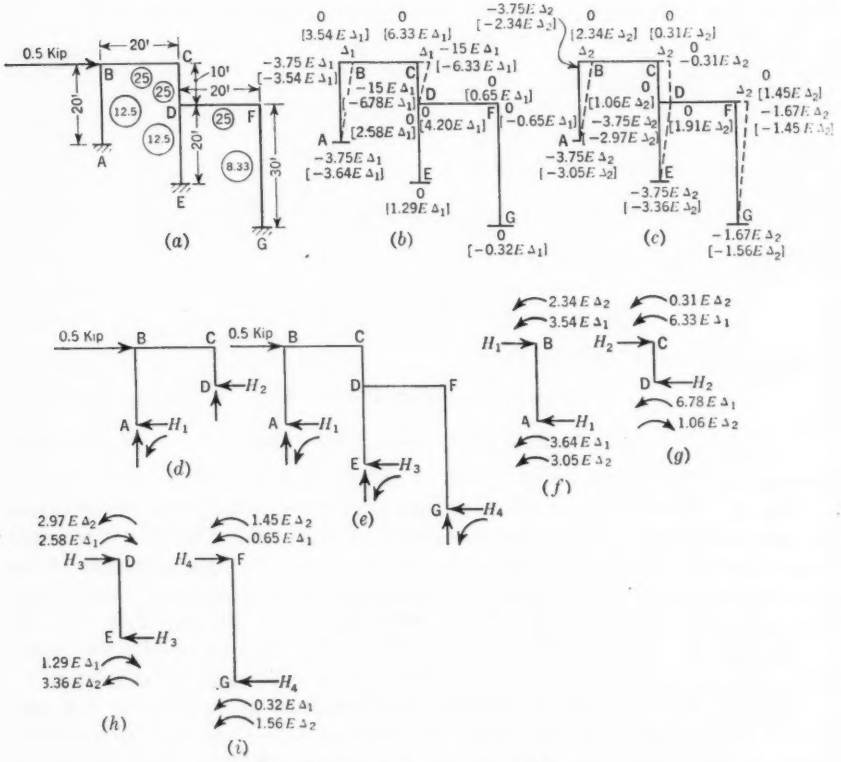


FIG. 5.—COMPLEX THREE-COLUMN BENT

Similarly, in Fig. 5(*g*),

$$H_2 = 1.311\ E\ \Delta_1 - 0.075\ E\ \Delta_2 \quad \text{.....(15b)}$$

In Fig. 5(*h*),

$$H_3 = -\ 0.193\ E\ \Delta_1 + 0.316\ E\ \Delta_2 \quad \text{.....(15c)}$$

and, in Fig. 5(*i*),

$$H_4 = 0.032\ E\ \Delta_1 + 0.100\ E\ \Delta_2 \quad \text{.....(15d)}$$

Substituting Eqs. 15 in Eqs. 14,

$$1.670\ E\ \Delta_1 + 0.194\ E\ \Delta_2 = 500 \quad \text{.....(16a)}$$

and

$$0.198\ E\ \Delta_1 + 0.685\ E\ \Delta_2 = 500 \quad \text{.....(16b)}$$

Solving Eqs. 16 simultaneously, $E\ \Delta_1 = 222$, and $E\ \Delta_2 = 666$.

Step 5. The numerical values of the moments caused by each deformation are determined by substituting the values of $E\,\Delta_1$ and $E\,\Delta_2$ in the moment terms shown in brackets in Figs. 5(*b*) and 5(*c*), and the following final moments in foot-pounds at the ends of all members are determined by summation: $M_{AB} = -2{,}838$; $M_{BA} = -2{,}345$; $M_{CD} = -1{,}612$; $M_{DC} = -799$; $M_{DE} = -1{,}407$; $M_{ED} = -1{,}954$; $M_{DF} = 2{,}203$; $M_{FG} = -1{,}109$; and $M_{GF} = -1{,}111$.

*Example 5. Vierendeel Truss.*—Fig. 6(*a*) represents a simple Vierendeel truss that supports a vertical load of 10 kips at the center of the chord BC. Values of $E$ and $I$ are assumed constant throughout.

Step 1. Three possible conditions of deformation exist—one for the transverse load on BC, and one for deformation of each panel of the structure. These are shown in Figs. 6(*b*), 6(*c*), and 6(*d*).

Step 2. Fixed-end moments are computed for the deformation conditions of Figs. 6(*b*), 6(*c*), and 6(*d*), and recorded as shown.

Step 3. The joints in Figs. 6(*b*), 6(*c*), and 6(*d*) are balanced and the final moments for each case are shown in brackets.

Step 4. To determine the values of $E\,R_1$ and $E\,R_2$, two equilibrium equations for the truss must be written. For example, in Fig. 6(*e*), from $\Sigma F_y = 0$,

$$V_1 + V_2 = 7.5 \qquad (17a)$$

and, in Fig. 6(*f*), from $\Sigma F_y = 0$,

$$V_3 + V_4 = 2.5 \qquad (17b)$$

In Fig. 6(*g*), from $\Sigma M_c = 0$,

$$V_1 = \frac{55.38 + 6.23\,E\,R_1 - 1.82\,E\,R_2}{12} \qquad (18a)$$

In Fig. 6(*h*), from $\Sigma M = 0$,

$$V_2 = \frac{1.70 + 6.23\,E\,R_1 - 1.82\,E\,R_2}{12} \qquad (18b)$$

In Fig. 6(*i*), from $\Sigma M = 0$,

$$V_3 = \frac{-8.30 + 1.82\,E\,R_1 - 6.23\,E\,R_2}{12} \qquad (18c)$$

In Fig. 6(*j*), from $\Sigma M = 0$,

$$V_4 = \frac{1.24 + 1.82\,E\,R_1 - 6.23\,E\,R_2}{12} \qquad (18d)$$

Substituting Eqs. 18 in Eqs. 17,

$$12.46\,E\,R_1 - 3.65\,E\,R_2 = 32.92 \qquad (19a)$$

and

$$3.64\,E\,R_1 - 12.46\,E\,R_2 = 37.06 \qquad (19b)$$

Solving Eqs. 19 simultaneously, $E\,R_1 = 1.94$; and $E\,R_2 = -2.43$. (Note that these are relative $R$-values corresponding to $R = 0$ for the vertical members of the frame. The moments obtained using these $R$-values will be correct, however, as can readily be seen by reference to the general slope-deflection equation.)

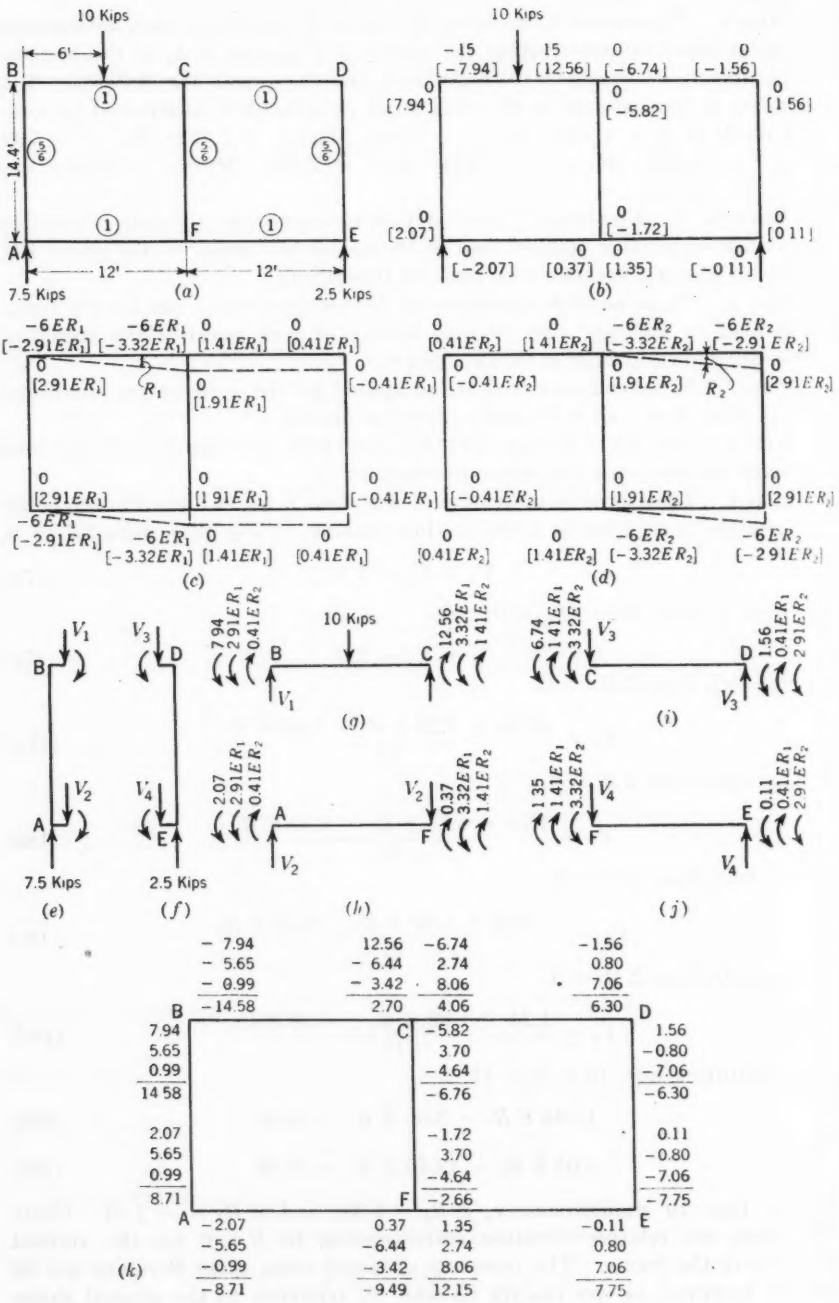


FIG. 6.—VIERENDEEL TRUSS

Step 5. The summation of the moments in Figs. 6(*b*), 6(*c*), and 6(*d*) and the final values of the moments in foot-kips at the ends of all members are shown in Fig. 6(*k*).

## Nonrectangular Frames

Nonrectangular or quadrangular frames, such as trapezoidal bents, gable bents, and Vierendeel trusses with inclined upper chords, are frequently used in practice. Their behavior under the action of applied forces which cause linear displacements of the joints (deformations of the quadrangles) is radically different from that of rectangular frames under similar conditions, as will be noted in the problems of this section.

*Relation Between Values of R for Members of a Quadrangle.*—The quadrangle represented in Fig. 7 is deformed by external forces (not shown) so that joints B and C move to positions B′ and C′, respectively. The member AD is

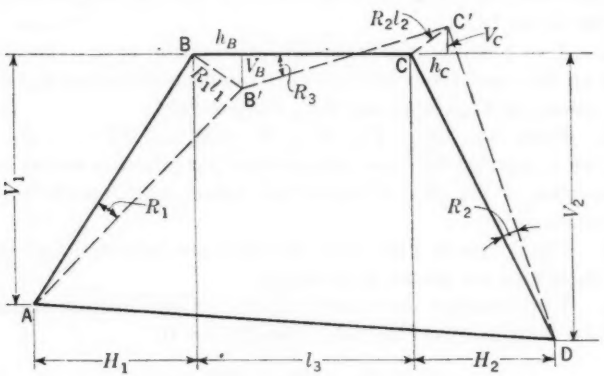


Fig. 7.—Deformation of a Quadrangle

assumed to remain fixed in direction during the displacements of B and C. It is desired to find the relations between the values of $R$ for AB, BC, and CD.

Let $l_1$, $l_2$, and $l_3$ represent the lengths of members AB, CD, and BC, respectively, and $R_1$, $R_2$, and $R_3$ the corresponding values of $R$. Let $H_1$ and $V_1$ represent the horizontal and vertical projections of AB, and $H_2$ and $V_2$ the horizontal and vertical projections of CD. Let $h_B$ and $v_B$ represent the horizontal and vertical components of the displacement of B, and $h_C$ and $v_C$ the horizontal and vertical components of the displacement of C.

From similar triangles, the following relations may be written: $\frac{h_B}{R_1 l_1} = \frac{V_1}{l_1}$; and $\frac{v_B}{R_1 l_1} = \frac{H_1}{l_1}$. Therefore, $h_B = R_1 V_1$, and $v_B = R_1 H_1$. Similarly, it may be shown that $h_C = R_2 V_2$, and $v_C = R_2 H_2$. Since $h_B = h_C$, $R_1 V_1 = R_2 V_2$; and

$$R_2 = \frac{R_1 V_1}{V_2} \qquad (20a)$$

Also, since the vertical displacement of C relative to B equals $-(v_B + v_C) = -(R_1 H_1 + R_2 H_2)$,

$$R_3 = \frac{-(R_1 H_1 + R_2 H_2)}{l_3} \quad \ldots\ldots (20b)$$

Eqs. 20 are the desired relations which enable one to express $R_2$ and $R_3$ for members CD and BC, respectively, in terms of $R_1$ for AB.

In the quadrangles of many frames, the projections $V_1$ and $V_2$ of AB and CD, Fig. 7, will be equal. Eqs. 20 may then be written:

$$R_2 = R_1 \quad \ldots\ldots (21a)$$

and

$$R_3 = \frac{-R_1 (H_1 + H_2)}{l_3} \quad \ldots\ldots (21b)$$

*Example 6. Trapezoidal Bent.*—Fig. 8(*a*) represents a trapezoidal bent (9*a*) with a horizontal load of 200 lb applied at B and an unsymmetrical vertical load of 1,000 lb on BC.

Step 1. Two possible conditions of deformation exist—one for the transverse load on BC, and the other for lateral deflection (sidesway) of the frame. These are shown in Figs. 8(*b*) and 8(*c*), respectively.

Step 2. From Eqs. 21 in Fig. 8(*c*), $R_2 = R_1$ and $R_3 = -R_1$. Fixed-end moments are computed for this deformation condition in terms of $E R_1$, and for the condition of Fig. 8(*b*) in numerical values, and recorded on the appropriate sketches.

Step 3. The joints in Figs. 8(*b*) and 8(*c*) are balanced and the final moments for each case are shown in brackets.

Step 4. To determine the value of $E R_1$, an equilibrium equation for the bent must be written. In Fig. 8(*d*), from $\Sigma F_x = 0$,

$$H_1 + H_2 = 200 \quad \ldots\ldots (22)$$

In Fig. 8(*e*), from $\Sigma M_C = 0$,

$$V_1 = 221.4 - 1.384\, E R_1 \quad \ldots\ldots (23a)$$

and, from $\Sigma M_B = 0$,

$$V_2 = 778.6 + 1.384\, E R_1 \quad \ldots\ldots (23b)$$

The values of $V_1$ and $V_2$ in Eqs. 23 are shown acting in Figs. 8(*f*) and (8*g*), respectively. In Fig. 8(*f*), from $\Sigma M = 0$,

$$H_1 = \frac{20.30\, E R_1 - 1{,}692}{15} \quad \ldots\ldots (24a)$$

and, in Fig. 8(*g*), from $\Sigma M = 0$,

$$H_2 = \frac{20.30\, E R_1 + 4{,}908}{15} \quad \ldots\ldots (24b)$$

Substituting Eqs. 24 in Eq. 22,

$$\frac{20.30\, E R_1 - 1{,}692}{15} + \frac{20.30\, E R_1 + 4{,}908}{15} = 200 \quad \ldots\ldots (25)$$

and solving Eq. 25, $E R_1 = -5.32$.

Step 5. Summation of the moments in Figs. 8(*b*) and 8(*c*) gives the following final moments: $M_{AB} = 228$; $M_{BA} = 428$; $M_{BC} = -428$; $M_{CB} = 640$; $M_{CD} = -640$; and $M_{DC} = -304$.

*Example 7. Two-Story Bent with Horizontal Loads.*—Fig. 9(*a*) represents a two-story bent with inclined legs which carries horizontal loads applied at joints B and E.

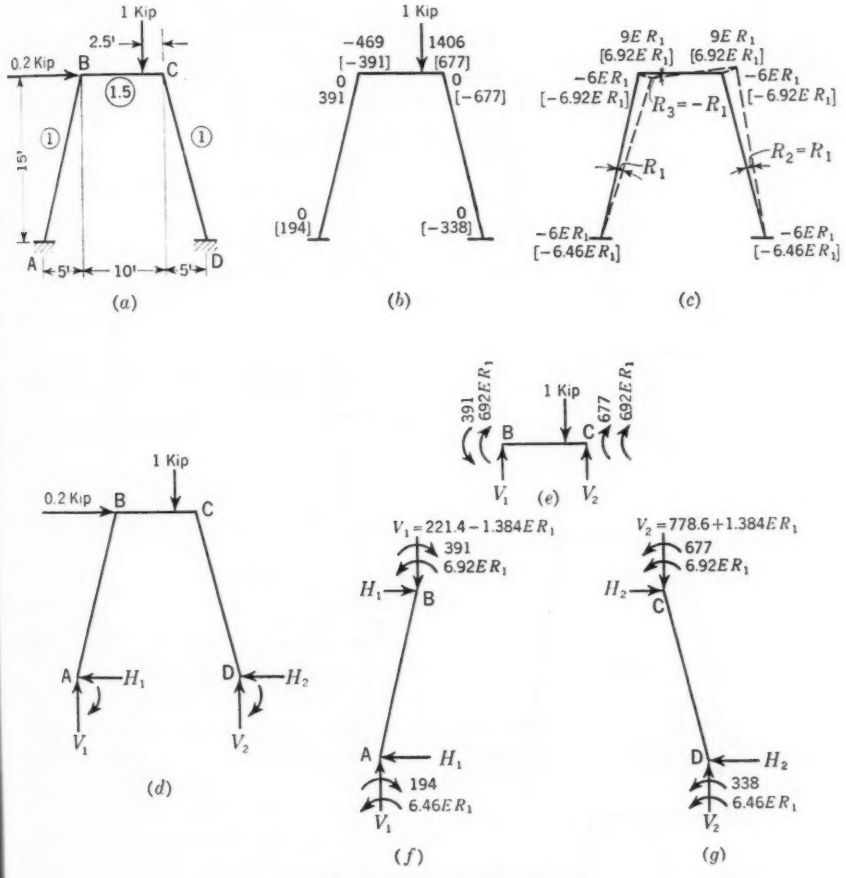


FIG. 8.—TRAPEZOIDAL BENT

Step 1. Two possible conditions of deformation exist—one for primary deformation of the first story which in turn causes secondary deformation of the second story, and the other for the deformation of the second story of the frame. These are shown in Figs. 9(*b*) and 9(*c*).

Step 2. By use of Eqs. 21, the *R*-values for all members in Fig. 9(*b*) have been expressed in terms of $R_1$ for AB, and for all members in Fig. 9(*c*) in terms of $R_2$ for BE. The corresponding fixed-end moments, in terms of $E\,R_1$ and $E\,R_2$, are recorded on these sketches.

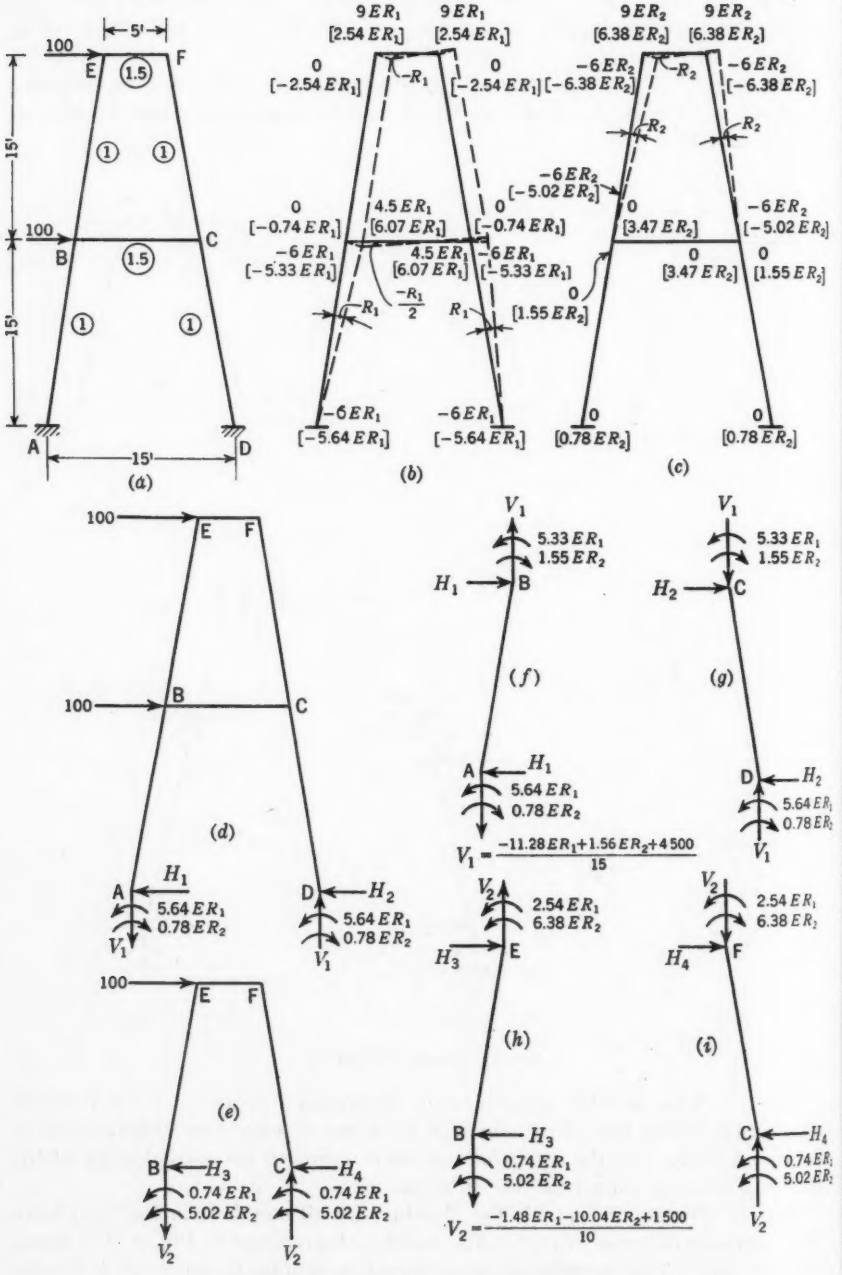


FIG. 9.—TWO-STORY BENT WITH HORIZONTAL LOADS

Step 3. The joints in Figs. 9(*b*) and 9(*c*) are balanced and the final moments for each case are shown in brackets.

Step 4. To determine the values of $E\,R_1$ and $E\,R_2$, two equilibrium equations for the bent must be written. In Fig. 9(*d*), from $\Sigma F_x = 0$,

$$H_1 + H_2 = 200 \qquad (26a)$$

and, in Fig. 9(*e*), from $\Sigma F_x = 0$,

$$H_3 + H_4 = 100 \qquad (26b)$$

In Fig. 9(*d*), from $\Sigma M_D = 0$,

$$V_1 = \frac{-\,11.28\,E\,R_1 + 1.56\,E\,R_2 + 4{,}500}{15} \qquad (27a)$$

and, in Fig. 9(*e*), from $\Sigma M_C = 0$,

$$V_2 = \frac{-\,1.48\,E\,R_1 - 10.04\,E\,R_2 + 1{,}500}{10} \qquad (27b)$$

The values of $V_1$ and $V_2$ in Eqs. 27 are shown acting in Figs. 9(*f*), 9(*g*), 9(*h*), and 9(*i*). In Figs. 9(*f*) and 9(*g*), respectively, from $\Sigma M = 0$,

$$H_1 = H_2 = \frac{54.54\,E\,R_1 - 12.42\,E\,R_2 + 4{,}500}{90} \qquad (28a)$$

In Figs. 9(*h*) and 9(*i*), respectively, from $\Sigma M = 0$,

$$H_3 = H_4 = \frac{11.64\,E\,R_1 + 35.56\,E\,R_2 + 1{,}500}{60} \qquad (28b)$$

Substituting Eqs. 28 in Eqs. 26,

$$\left.\begin{aligned} 54.54\,E\,R_1 - 12.42\,E\,R_2 &= 4{,}500 \\ 11.64\,E\,R_1 + 35.56\,E\,R_2 &= 1{,}500 \end{aligned}\right\} \qquad (29)$$

Solving Eqs. 29 simultaneously, $E\,R_1 = 85.7$ and $E\,R_2 = 14.1$.

Step 5. The numerical values of the moments in Figs. 9(*b*) and 9(*c*) may now be calculated, and the following moments at the ends of all members are determined by summation: $M_{AB} = M_{DC} = -472$; $M_{BA} = M_{CD} = -434$; $M_{BE} = M_{CF} = -134$; $M_{EB} = M_{FC} = -308$; $M_{BC} = M_{CB} = 569$; and $M_{EF} = M_{FE} = 308$.

*Example 8. Gable Bent.*—Fig. 10(*a*) represents a gable bent with a horizontal load of 1,000 lb applied at joint B.

Step 1. Two possible conditions of deformation exist—one for deformation of EDCB, and the other for linear displacement of B which in turn causes an equal displacement of D (an alternate deformation of ABCD might be used instead). These are shown in Figs. 10(*b*) and 10(*c*), respectively.

Step 2. By use of Eqs. 20, the $R$-values for all members in Fig. 10(*b*) have been expressed in terms of $R_1$ for ED, and the $R$-value of ED in Fig. 10(*c*) has been expressed in terms of $R_2$ for AB. The corresponding fixed-end moments, in terms of $E\,R_1$ and $E\,R_2$, are recorded on these sketches.

Step 3. The joints in Figs. 10(*b*) and 10(*c*) are balanced and the final moments for each case are shown in brackets.

Step 4. To determine the values of $E\,R_1$ and $E\,R_2$, two equilibrium equations for the bent must be written.

In Fig. 10(*d*), from $\Sigma F_x = 0$,

$$H_1 + H_3 = 1{,}000 \quad \ldots\ldots (30a)$$

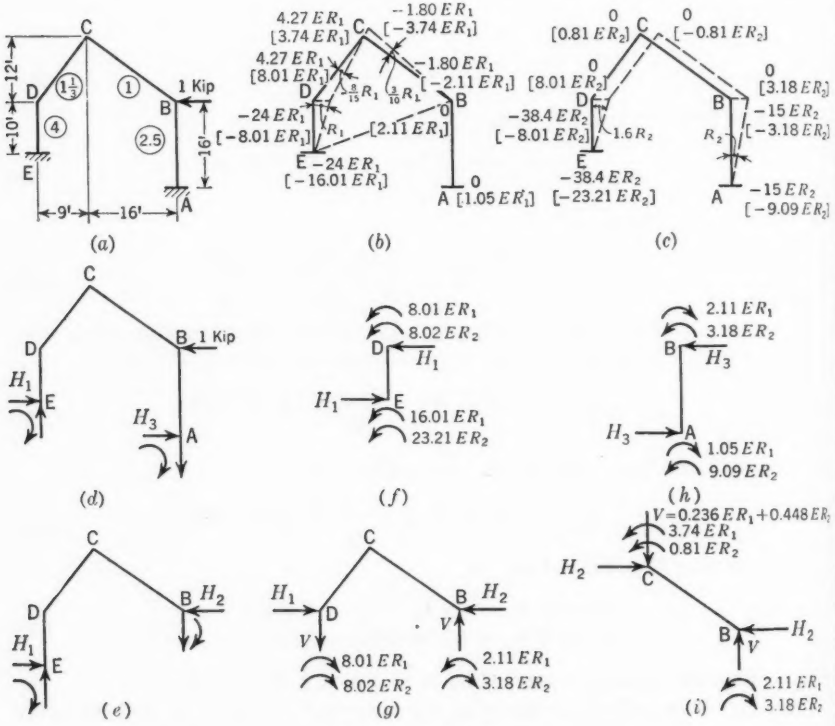


FIG. 10.—GABLE BENT

and, in Fig. 10(*e*), from $\Sigma F_x = 0$,

$$H_1 - H_2 = 0 \quad \ldots\ldots (30b)$$

In Fig. 10(*f*), from $\Sigma M = 0$,

$$H_1 = -2.402\,E\,R_1 - 3.122\,E\,R_2 \quad \ldots\ldots (31a)$$

in Fig. 10(*g*), from $\Sigma M = 0$,

$$H_3 = 0.198\,E\,R_1 - 0.767\,E\,R_2 \quad \ldots\ldots (31b)$$

and, in Fig. 10(*h*), from $\Sigma M_B = 0$,

$$V = 0.236\,E\,R_1 + 0.448\,E\,R_2 \quad \ldots\ldots (31c)$$

The value of $V$ in Eq. 31c is shown acting in Fig. 10(*i*) in which, from $\Sigma M_B = 0$,

$$H_2 = 0.802\,E\,R_1 + 0.400\,E\,R_2 \quad \ldots\ldots (31d)$$

Substituting Eqs. 31 in Eqs. 30

$$\left.\begin{aligned} -2.204\,E\,R_1 - 3.889\,E\,R_2 &= 1{,}000 \\ -3.204\,E\,R_1 - 3.522\,E\,R_2 &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (32)$$

Solving Eqs. 32 simultaneously, $E\,R_1 = 748$; and $E\,R_2 = -680$.

Step 5. The numerical values of the moments in Figs. 10(*b*) and 10(*c*) may now be computed, and the final moments at the ends of all members are determined by summation: $M_{AB} = 6{,}970$; $M_{BA} = 3{,}740$; $M_{BC} = -3{,}740$; $M_{CB} = -2{,}250$; $M_{CD} = 2{,}250$; $M_{DC} = 550$; $M_{DE} = -550$; and $M_{ED} = 3{,}800$.

## Conclusions

In this paper a method of procedure has been developed whereby the principle of superposition is utilized in conjunction with the slope-deflection or moment distribution method in the analysis of rigid frames.

The following conclusions are based on the foregoing analyses:

1. The method of procedure is general, and the results obtained are exact for the assumptions commonly made in rigid frame analysis.
2. Although many computations are required in the analysis of a frame having a large number of deformation conditions, the calculations are simple and all operations of multiplication and division may usually be performed with sufficient accuracy on a 10-in. slide rule.
3. At all times special care must be given to the correct formulation of equilibrium equations and, in the case of nonrectangular frames, to the determination of the relative *R*-values for the members of the quadrangles of the frame being considered.
4. For frames of unusual proportions where convergence by the moment distribution method is slow, it is recommended that the slope-deflection method be used to balance the moments at the joints in step 3 of the outline of method of procedure.
5. The principle of superposition is an indispensable aid in the analysis of rigid frames of all types in which applied loads cause linear displacements of the joints.

## APPENDIX

### Bibliography

The following is a selected list of references which contain material related to the subject matter of this paper:

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(2) **"Analysis of Continuous Frames by Distributing Fixed-End Moments,"** by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), pp. 1–156.

(3) **"Continuous Frames of Reinforced Concrete,"** by Hardy Cross and N. D. Morgan, John Wiley & Sons, Inc., New York, N. Y., 1932.

(4) **"Wind Stresses by Slope Deflection and Converging Approximations,"** by John E. Goldberg, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), pp. 962–985.

(5) **"Theory of Modern Steel Structures,"** by L. E. Grinter, Vol. 2, The Macmillan Company, New York, N. Y., 1937. (*a*) p. 160.

(6) **"The Theory and Practice of Modern Framed Structures,"** by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, Pt. II, Statically Indeterminate Structures and Secondary Stresses, John Wiley & Sons, Inc., New York, N. Y., 1929.

(7) **"The Analysis of Vierendeel Trusses by Successive Approximations,"** by L. C. Maugh, *Memoirs*, International Assn. for Bridge and Structural Engrs., Vol. 3, 1935, pp. 333–354.

(8) **"A Rapid and Concise Method of Analyzing Rigid Viaduct Bents,"** by L. C. Maugh, *Engineering News-Record*, Vol. 114, No. 11, March 14, 1935, p. 379.

(9) **"Elementary Treatise on Statically Indeterminate Stresses,"** by John Ira Parcel and George Alfred Maney, John Wiley & Sons, Inc., New York, N. Y., 1936. (*a*) 2d Ed., p. 226.

(10) **"Simultaneous Equations in Mechanics Solved by Iteration,"** by W. L. Schwalbe, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 939–969.

(11) **"Elastic Energy Theory,"** by J. A. Van den Broek, John Wiley & Sons, Inc., New York, N. Y., 2d Ed., 1942.

(12) **"Successive Elimination of Unknowns in the Slope Deflection Method,"** by John B. Wilbur, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 346–371.

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(14) **"Analysis of Vierendeel Trusses,"** by Dana Young, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 869–938.

(15) **"Analysis of Quadrangular Frames by the Moment Distribution Method,"** by David M. Wilson, thesis presented to the Univ. of Michigan, Ann Arbor, in March, 1937, in partial fulfilment of the requirements for the degree of Civil Engineer.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

# SIMULTANEOUS CONSOLIDATION OF CONTIGUOUS LAYERS OF UNLIKE COMPRESSIBLE SOILS

By Hamilton Gray,[1] Jun. Am. Soc. C. E.

Synopsis

The results of an analysis of a condition which is sometimes encountered in foundation investigations—the consolidation of two contiguous layers of clay possessing different consolidation characteristics—are presented in this paper. A special case of such a phenomenon exists when one of two contiguous layers is relatively incompressible and functions merely to impede the outflow of water from the compressible stratum. When two adjoining compressible strata are made to consolidate under an applied load, the behavior of each stratum is influenced by the presence and action of the other. The extent of the mutual interference or influence of the layers depends on the physical properties (compressibility and permeability) of the materials comprising each layer. The analysis determines quantitatively how these physical properties govern the simultaneous consolidation of the two layers.

The details of the analysis have been placed on file for reference in Engineering Societies Library[1a] in New York, N. Y. They have been omitted from this paper, since they are long and do not influence the application of the results. The basic assumptions upon which the analysis is predicated are given in this paper, together with the appropriate differential equations and the final results. The user is thus able to apply these solutions with an understanding of the restrictions under which they have been secured. Graphs are presented illustrating the results secured for specific conditions. The nature of these solutions is in entire agreement with what "physical intuition" might anticipate, and a comparison is made between various approximations that have been made heretofore, and the "rigorous" solutions herein outlined.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **July 1, 1944.**

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The results are purely theoretical and must be so considered until comparison with observed phenomena in the field confirms or denies their applicability to engineering problems. They are presented in the expectation that such comparisons may be more numerous if the nature of the theoretical approach is known.

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## INTRODUCTION

The original theory of the consolidation of clay deposits[2] was offered by Karl Terzaghi, M. Am. Soc. C. E., in 1925. Considerable interest has been shown recently in extensions of this theory to the more complicated practical problems involving three-dimensional flow and compression[3,4,5,6] and stratified clay deposits.[7] Definite progress has been made in developing solutions for certain of these problems. A solution of the one-dimensional consolidation problem involving two contiguous layers of unlike clays has not appeared heretofore, although the technique required for obtaining it has been available for some years[8,9] and somewhat analogous problems have been treated in the theory of heat flow.

The purpose of this paper is to draw attention to this solution and to indicate the nature of the results obtained. It must be realized that the application of these more recent developments would be justifiable only in the case of relatively large and expensive structures, since the analyses appear rather forbidding and at best necessitate considerable time and effort. Nonetheless, in connection with important structures it is frequently desirable to apply such advanced theories to secure one of perhaps a number of rough estimates of the possibilities inherent to the situation. Today engineers in increasing numbers are drawing extensively on the results of mathematical analysis as an aid in their work. In some cases the answer so obtained may be criticized as little better than a guess, but in any event it is a relatively "intelligent" or "conditioned" guess and, if actual behavior in the field is observed properly, one will ultimately be in a position to decide whether the "guess" was worth the effort or not. It is well to emphasize that experimental verification of the theories currently utilized for predicting the settlement of structures, perhaps by models, but more particularly by observing full-size structures, has not been as common as is desirable and that unwarranted confidence is sometimes placed in estimates that are based on theory unsubstantiated by field observation. Frequently

[2] "Erdbaumechanik auf bodenphysikalischer grundlage," by K. Terzaghi, F. Deuticke, Vienna, 1925.

[3] "General Theory of Three-Dimensional Consolidation," by Maurice A. Biot, *Journal of Applied Physics*, Vol. 12, 1941, No. 2, p. 155.

[4] "Consolidation Settlement Under a Rectangular Load Distribution," by M. A. Biot, *ibid.*, No. 5, p. 426.

[5] "Consolidation Settlement of a Soil with an Impervious Top Surface," by M. A. Biot and F. M Clingan, *ibid.*, No. 7, p. 578.

[6] "Bending Settlement of a Slab Resting on a Consolidating Foundation," by M. A. Biot and F. M Clingan, *ibid.*, Vol. 13, 1942, No. 1, p. 35.

[7] "Factors Affecting the Accuracy of Settlement Rate Forecasts," by G. P. Tschebotareff, *Proceedings*, 18th Annual Meeting, Highway Research Board, Pt. II, 1938, p. 120.

[8] "The Cooling of a Solid Sphere with a Concentric Core of a Different Material," by H. S. Carslaw, *Proceedings of the Cambridge Philosophical Society*, Vol. XX, 1921, p. 399.

[9] "Bromwich's Method of Solving Problems in the Conduction of Heat," by H. S. Carslaw, *London, Edinburgh and Dublin Philosophical Magazine and Journal of Science*, Series 6, Vol. 39, 1920, p. 603.

actual conditions invalidate some of the assumptions on which a theory is based, and failure to consider this possibility may then result in large discrepancies between predicted and observed behavior. The following developments are offered as "tools," the accuracy of which remains to be determined, of which the engineer concerned with foundation design may avail himself at his discretion.

## A Special Case of Impeded Drainage

By way of introduction the relatively simple problem of a compressible stratum with impeded drainage will be considered. This is actually a special case of the more general problem treated herein, but is introduced here because of the relative ease with which it can be analyzed. The method has long been known;[10] it represents an extension of the usual attack by means of differential equations and Fourier series. Two practical applications are found when one investigates (*a*) the error involved in the assumption of "free outflow" from the surface of a clay sample being compressed in the laboratory under known conditions, and (*b*) the case of a relatively thin but compressible soil layer overlain by a thick bed of incompressible and relatively impervious soil.

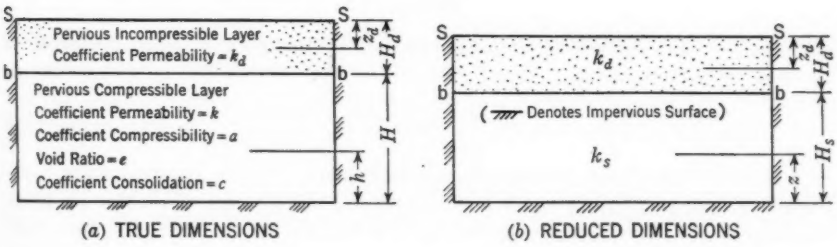


Fig. 1

The problem is shown schematically in Fig. 1, which serves also to define certain of the letter symbols introduced in the analysis. The remaining symbols are defined as they appear in the text and the complete list is presented, for the convenience of the reader, in the Appendix. The "reduced" dimensions of Fig. 1(*b*) were introduced for mathematical simplicity and are related to the true dimensions by the equations:

$$H_s = \frac{H}{1+e} \qquad (1a)$$

and

$$z = \frac{h}{1+e} \qquad (1b)$$

In accordance with the usual custom in dealing with the consolidation problem, it is assumed that the voids of both compressible and incompressible layers are completely filled with an incompressible liquid (water). It is further

[10] "An Elementary Treatise on Fourier's Series and Spherical, Cylindrical, and Ellipsoidal Harmonics," by W. E. Byerly, Ginn & Co., Boston, 1893, pp. 117–121.

assumed that, before the consolidation process is initiated, the pressure, $u_z$, in the void liquid varies according to the law

$$u_z = \gamma z_d \quad \ldots\ldots (2)$$

in which $\gamma$ is the density of the liquid, and that at all times the liquid pressure is atmospheric when $z_d = 0$. A normal stress of intensity, $p$, is abruptly applied to the surface S-S, resulting in an upward flow of water associated with the compression or void reduction of the lower layer under the influence of this added load. If $u_d$ (commonly referred to as "hydrostatic excess pressure") denotes the fluid pressure, in excess of that given by Eq. 2, induced by $p$ in the incompressible layer, the flow of liquid that occurs in this incompressible layer is governed by LaPlace's equation:[11]

$$\frac{\partial^2 u_d}{(\partial z_d)^2} = 0 \quad \ldots\ldots (3a)$$

Similarly, the excess fluid pressure, $u$, in the compressible layer varies in accordance with the formula[2]

$$c \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \quad \ldots\ldots (3b)$$

in which $c$, the coefficient of consolidation of the compressible material, is defined by the equation

$$c = \frac{k}{a \gamma (1 + e)} \quad \ldots\ldots (4)$$

$- a$ is the slope of the pressure, void-ratio curve; $+ a$ is known as the coefficient of compressibility; and $t$ is the time. The solution of the problem consists of two expressions—one for $u_d$, as a function of $z_d$ and $t$, and the other for $u$, as a function of $z$ and $t$. These expressions must satisfy the differential Eqs. 3 and, in addition, the following "boundary conditions":
When $z = 0$—

$$\frac{\partial u}{\partial z} = 0 \quad \ldots\ldots (5a)$$

when $t = 0$—

$$u = p \text{ (a constant)} \quad \ldots\ldots (5b)$$

when $z_d = 0$

$$u_d = 0 \quad \ldots\ldots (5c)$$

and, when $z_d = H_d$ and $h = H$—that is, at the common boundary—

$$u = u_d; \quad \text{and} \quad k \frac{\partial u}{\partial h} + k_d \frac{\partial u_d}{\partial z_d} = 0 \quad \ldots\ldots (5d)$$

Since $h = z (1 + e)$, Eq. 5*d* may be written

$$\left[ k_s \frac{\partial u}{\partial z} \right]_{z=H_s} + \left[ k_d \frac{\partial u_d}{\partial z_d} \right]_{z_d=H_d} = 0 \quad \ldots\ldots (6)$$

[11] "The Flow of Homogeneous Fluids Through Porous Media," by M. Muskat, 1st Ed., McGraw-Hill Book Co., Inc., New York, 1937.

in which $k_s = \frac{k}{1+e}$. By differentiation and substitution it is readily shown that—

$$u = 2\,p \sum_{n=1}^{n=\infty} \epsilon^{-TN} \left[ \frac{\sin r_n \cos r_n \frac{z}{H_s}}{r_n + \frac{1}{2} \sin 2\,r_n} \right] \quad \ldots\ldots\ldots\ldots\ldots (7a)$$

and

$$u_d = p \frac{z_d}{H_d} \sum_{n=1}^{n=\infty} \epsilon^{-TN} \left( \frac{\sin 2\,r_n}{r_n + \frac{1}{2} \sin 2\,r_n} \right) \ldots\ldots\ldots\ldots\ldots (7b)$$

—are two functions which, respectively, satisfy the differential Eqs. 3 and the prescribed boundary conditions, provided $r_n$ is a root of the equation—

$$r \tan r = \frac{H_s}{H_d} \frac{k_d}{k_s} = R \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8a)$$

In Eqs. 7, $\epsilon$ is the base of natural logarithms and the symbols in the exponent

$$N = (r_n)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8b)$$

and

$$T = \frac{c\,t}{(H_s)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8c)$$

are used solely to simplify typography. In Eq. 8c the symbol $T$ is a dimensionless quantity commonly termed a "time factor."

The degree of consolidation at any point in the compressible stratum is defined as

$$U_p = 1 - \frac{u}{p} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

and the average degree of consolidation for the entire stratum is then given by

$$U = \frac{1}{H_s} \int_0^{H_s} U_p\,dz = 1 - \frac{1}{p\,H_s} \int_0^{H_s} u\,dz \ldots\ldots\ldots\ldots (10)$$

The degree of consolidation, $U$, equals the ratio of current subsidence or compression to the ultimate values of these quantities under the existing pressure provided the relation between void ratio and intergranular pressures in the soil is linear. Substituting the expression Eq. 7*a* for $u$ under the integral sign in Eq. 10, and evaluating:

$$U = 1 - 2 \sum_{n=1}^{n=\infty} \frac{\epsilon^{-TN}}{r_n} \frac{\sin^2 r_n}{r_n + \frac{1}{2} \sin 2\,r_n} \ldots\ldots\ldots\ldots (11)$$

The degree of consolidation is frequently expressed as a percentage and written as $U\%$. The values of $U\%$ then are one hundred times the magnitudes of $U$ given by Eq. 11. As $k_d$ approaches infinity (corresponding to unimpeded or free outflow from the upper surface of the compressible layer), $R$ likewise becomes infinite and the roots of Eq. 8*a* become simply odd multiples of $\frac{\pi}{2}$, and may be represented by

$$r_n = (2\,m + 1)\frac{\pi}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which $m$ takes on successive integer values beginning with $m = 0$. The corresponding equation for $U$ is

$$U = 1 - \frac{8}{\pi^2} \sum_{m=0}^{m=\infty} \frac{1}{(2\,m + 1)^2}\, \epsilon^{-TN'} \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

which coincides with the familiar solution obtained when the boundary condition Eq. 5*d* for the compressible layer is $u = 0$ when $z = H_s$. In Eq. 13, the exponent $N'$ is substituted for

$$N' = \frac{(2\,m + 1)^2\, \pi^2}{4} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

to simplify typography. For fixed values of $H_s$, $H_d$, $k_s$, and $k_d$, $R$ is determined, and the roots of Eq. 8*a* may be found by the method of successive approximations. Typical curves such as those illustrated in Fig. 2, showing the variation of $U$ with $T$ for different values of $R$, may then be plotted.

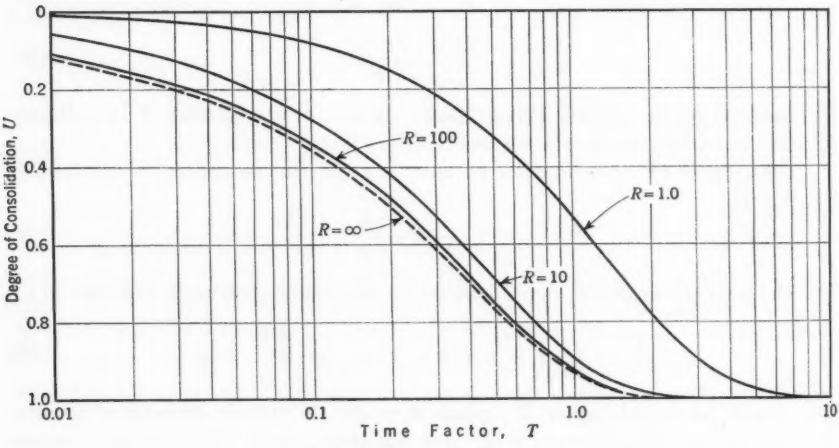


FIG. 2.—CURVES OF $U$ VERSUS $T$

For large values of $T$, only one term of the series for $U$ is needed for sufficient accuracy in plotting the curves. Hence, to find the approximate value of $T > 0.5$ required for a given value of $U > 0.76$ when $R$ is known, it is sufficient to use the equation:

$$U = 1 - \frac{2 \sin^2 r_1}{r_1 + \frac{1}{2} \sin 2\, r_1} \frac{\epsilon^{-NT}}{r_1} \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Solving for $T$ (as in Eqs. 7, $N = (r_n)^2$, the value of $n$ being 1 in this case):

$$T = \frac{1}{(r_1)^2} \log \frac{2 \sin^2 r_1}{(1 - U)\, r_1\, (r_1 + \frac{1}{2} \sin 2\, r_1)} \ldots\ldots\ldots\ldots (16)$$

in which $r_1$ is the smallest root of $r \tan r = R$ (see Eq. 8*a*).

In determining the value of $k$ from the results of a consolidation test it is necessary to utilize the relationships:

$$c = (H_s)^2 \frac{T}{t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17a)$$

and

$$k = a\,\gamma\,c\,(1 + e) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17b)$$

Eq. 17*b* may also be written

$$k_s = a\,\gamma\,c = a\,\gamma\,(H_s)^2 \frac{T}{t} \ldots\ldots\ldots\ldots\ldots\ldots (17c)$$

In these formulas $a$ is determined from the pressure, void-ratio diagram, and $H_s$ by actual measurements. It is customary to compare the shape of an empirical time-consolidation curve of the tested soil with that of the theoretical curve, $R = \infty$, of Fig. 2. These two curves are usually rather similar in shape, but the empirical curve is plotted to scales of elapsed time, $t$, and measured compression of the soil sample, whereas the curve, $R = \infty$, of Fig. 2 is plotted to the scales of the dimensionless quantities $T$ and $U$. It is generally possible to determine, for most empirical curves, the approximate value of elapsed time, $t$, which is required to attain some specified portion, say 60% or 90%, of the ultimate compression under the prevailing load.[12,13] To be specific, the elapsed time required to reach 90% of the ultimate compression, when drainage is unobstructed, will be denoted by $t_9$. The corresponding value of $U$ is 0.9, and of $T$, 0.848. Then

$$k_s = a\,\gamma\,(H_s)^2 \frac{0.848}{t_9} \ldots\ldots\ldots\ldots\ldots\ldots (18a)$$

and

$$k = k_s\,(1 + e) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18b)$$

in which $e$ is the average void ratio under the load increment which resulted in the empirical time-consolidation curve, and $a$ is determined from the pressure, void-ratio diagram.

This value of $k$ will be accurate only if the actual value of $R$ is infinite. If $R \neq \infty$, consolidation will occur in accordance with Eq. 11 rather than Eq. 13. The empirical curve should then be compared with a theoretical curve for $R \neq \infty$ rather than with the theoretical curve for $R = \infty$. However, the correct value of $R$ is unknown since it depends upon the value of $k$ to be determined.

It is possible to avoid this dilemma as follows: Assuming that $R \neq \infty$, let $t_a$ denote the observed time which corresponds to 90% of the ultimate compression given by the empirical curve. Since drainage is impeded, $t_a > t_9$. Computing as before by using $T = 0.848$:

$$k_{sa} = a\,\gamma\,(H_s)^2 \frac{0.848}{t_a} \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

in which $k_{sa}$ is the "apparent" permeability of the soil. Since $t_a > t_9$,

[12] "Principles of Final Soil Classification," by Charles Terzaghi, *Public Roads*, Vol. 8, May, 1927, p. 41.
[13] "Improved Soil Testing Methods," by G. Gilboy, *Engineering News-Record*, May 21, 1936, p. 732.

$k_{sa} < k_s$. Had the actual value of $R$ been known, a magnitude of $T$ greater than 0.848 could have been used, this magnitude being found by means of Eq. 16, in which $U = 0.9$. Use of this correct value of $T$ (denoted by $T_a$) would have given the result, $k_s$, at once. Since $\frac{T_a}{0.848}$ may be determined as a function of $R$, when $U = 0.9$, by means of Eq. 16, the ratios $\frac{k_s}{k_{sa}} = \frac{t_a}{t_9} = \frac{T_a}{0.848}$ may also be found in terms of $R$ (Eq. 8$a$). If $M = \frac{H_s}{H_d} k_d$, $\frac{M}{k_{sa}} = R \frac{t_a}{t_9}$. For any chosen value of $R$, the ratios $\frac{t_a}{t_9}$ and $\frac{k_s}{k_{sa}}$ can be found, and thence, $\frac{M}{k_{sa}}$. A plot of corresponding values of $\frac{k_s}{k_{sa}}$ versus $\frac{M}{k_{sa}}$ is shown in Fig. 3.

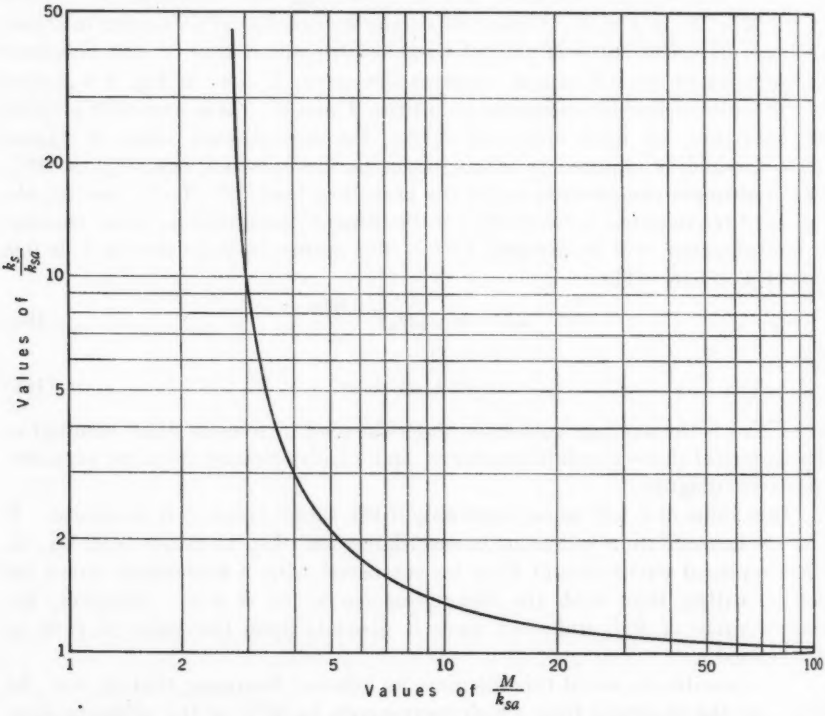


FIG. 3.—THE RATIO OF TRUE PERMEABILITY TO APPARENT PERMEABILITY AS A FUNCTION OF $\frac{M}{k_{sa}}$ (FOR $U = 0.9$)

From this curve it appears that, when $R = 100$, the ratio of $\frac{k_s}{k_{sa}}$ is only 1.025 so that it may be concluded that the drainage medium exercises a negligible influence if the value of $R$ exceeds 100. Ordinarily the drainage media used in laboratory testing are of such a nature that $R$ is of the order of magnitude of $10^5$. If, in any instance, the value of $R$ should fall below 100,

the curve of Fig. 3 would permit proper correction to be made in determining the value of $k_s$.

Another application of the results expressed in Eqs. 8*a* and 9 is involved in the subsidence of a load placed upon a bed of relatively incompressible soil of low permeability, such as a cohesionless silt, which overlies a stratum of compressible clay. It is assumed that the load exerts a uniform pressure on the soil and that its lateral extent is so great as to afford no appreciable reduction in pressure increase between the upper surface of the silt and the bottom of the clay, which is assumed to rest on impervious rock; that is, a load of 1 ton per sq ft applied at the ground surface creates an additional pressure of nearly the same magnitude at the rock surface, and at all intermediate elevations.

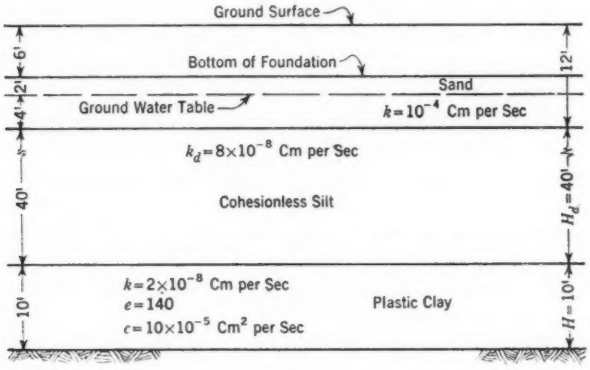


FIG. 4

The consolidation phenomenon then is one-dimensional for all practical purposes. Suppose that subsoil and laboratory investigations have established data shown in Fig. 4. Eq. 8*a* yields

$$R = \frac{k_d}{k_s}\frac{H_s}{H_d} = \frac{80 \times 10^{-9}}{8.33 \times 10^{-9}}\frac{4.16}{40.0} = 1.0.$$

The time required to reach any fraction of the ultimate subsidence of approximately 4 in. may be secured by means of the curve $R = 1.0$ in Fig. 2. For example, 3 in. of settlement corresponds to $U = 75\%$ and $T = 1.85$ according to this curve. The time in years is obtained from

$$t = T\frac{(H_s)^2}{c} \times \frac{1}{3.16 \times 10^7} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

in which $3.16 \times 10^7$ is the number of seconds in a year. Thus,

$$t = 1.85\frac{(30.48 \times 4.16)^2}{10 \times 10^{-5}} \times \frac{1}{3.16 \times 10^7} = \frac{1.85}{3.16}(3.048 \times 4.16)^2 = 9.5 \text{ yr.}$$

Had the permeability of the silt been of the order of $10^{-4}$ cm per sec the value

of $R$ would have been

$$\frac{10^{-4}}{8.33 \times 10^{-9}} \frac{4.16}{44} = \frac{41.6}{44} \times \frac{10}{8.33} \times 10^3 = 1.137 \times 10^3 = 1{,}137.$$

Using the curve $R = \infty$ in Fig. 2, the value $T = 0.48$ corresponds to $U = 75\%$; whence $t = 9.5 \times \frac{0.48}{1.85} = 2.46$ yr.

It should be realized that the drainage conditions affect only the time rate of compression and have no influence on the total amount of compression. Therefore consideration of the drainage conditions is required only when the settlement rate is of importance.

Another application of the solution given by Eq. 7*a* consists of the computation of the variation, brought about by consolidation, of the shearing strength of a clay. In some instances it is feasible to schedule construction operations for earth dams or embankments in such a manner that a weak foundation, originally incapable of supporting the finished structure, may be gradually loaded and compressed during the construction period so that its strength is never exceeded, but gradually increases as the void ratio is reduced. Thus, ultimately, it is able to support the finished structure safely. In such circumstances it is necessary to know not only the shearing, but also the consolidation characteristics of the foundation material. If the soil, which is to be strengthened by consolidation, does not have free drainage, the usual consolidation theory based on the assumption of unimpeded drainage will yield results that are on the unsafe side.

In considering the variation in shearing strength one may deal with the average strength of the weak stratum, in which case curves similar to those of Fig. 2 are useful, or one may deal with the weakest zone in the consolidating stratum. The shearing resistance at any point is a function primarily of the void ratio which may be found by means of the pressure, void-ratio diagram used in conjunction with Eq. 7*a*. At any instant of time the value of $u$ is greatest at the bottom of the consolidating stratum, see Fig. 1, where $z = 0$. At this elevation the void ratio is greater and the shearing strength is less than for values of $z > 0$. Setting $z = 0$, Eq. 7*a* becomes

$$u = 2\,p \sum_{n=1}^{n=\infty} \frac{\sin r_n}{r_n + \frac{1}{2} \sin 2\,r_n} \epsilon^{-T N} \quad \ldots\ldots\ldots\ldots\ldots (21)$$

If the thickness and the coefficient of consolidation of the consolidating stratum are known, $T = \frac{c\,t}{(H_s)^2}$ (Eq. 8*b*), and $u$ may be expressed as a function of the time lapse following the application of a load increment, $p$. Therefore, one is in a position to determine the variation of shear strength with the passage of time at the elevation where the stratum is weakest. In this connection it is convenient to plot $u$ as a function of $T$ in accordance with Eq. 21.

### The General Case of Two Adjacent Compressible Strata

In analyzing the problem of two contiguous layers of consolidating soils, it is expedient to consider the layers in a manner similar to that in the preceding

text. This problem is depicted in Fig. 5, there being two distinct cases to consider—(*a*) free drainage top and bottom, and (*b*) drainage at top only, as shown. In Fig. 5 all the dimensions shown are "reduced." The layers of

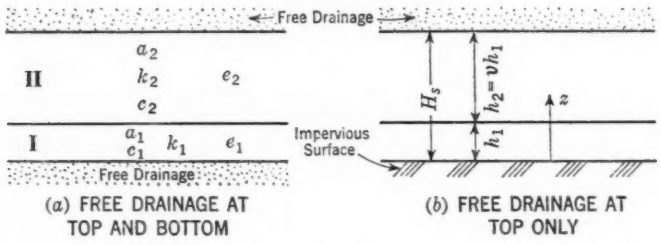


FIG. 5

soil are numbered I and II, and the corresponding soil coefficients are identified by appropriate subscripts, 1 and 2. For each material,

$$k = a\, c\, \gamma\, (1 + e) \qquad (22)$$

In addition, the following transformations are convenient:

$$k_{s1} = \frac{k_1}{1 + e_1} \qquad (23a)$$

$$k_{s2} = \frac{k_2}{1 + e_2} \qquad (23b)$$

and the introduction of four dimensionless numbers—

$$\mu^2 = \frac{c_I}{c_2}; \qquad \sigma = \frac{1}{\mu}\frac{k_{s1}}{k_{s2}}; \qquad \nu = \frac{h_2}{h_1}; \qquad \text{and} \qquad T = \frac{c_1}{(h_1)^2}\, t \qquad (24)$$

—is useful. The last ($T$) is the "time factor" for this case.

It is possible to solve the two basic cases simultaneously, and, in summarizing, the boundary conditions have been arranged in Table 1 so as to emphasize the similarity. For the lower layer the law of fluid flow may be expressed by

$$c_1 \frac{\partial^2 u_1}{\partial z^2} = \frac{\partial u_1}{\partial t} \qquad (25a)$$

and for the upper layer the corresponding equation is

$$c_2 \frac{\partial^2 u_2}{\partial z^2} = \frac{\partial u_2}{\partial t} \qquad (25b)$$

in which $u_1$ and $u_2$ denote the fluid pressures in excess of hydrostatic in the lower and upper layers, respectively.

Boundary condition 5, Table 1, is more restrictive than is necessary, since solutions can be obtained in cases where $u$ is any function of $z$ when $t = 0$. It may be shown by direct substitution that, for the case of drainage at both top

TABLE 1.—BOUNDARY CONDITIONS

| No. | Boundary | Drainage top and bottom | Drainage top only |
|---|---|---|---|
| 1 | $z = 0$ | $u_1 = 0$ | $\frac{\partial u_1}{\partial z} = 0$ |
| 2 | $z = H$ | $u_2 = 0$ | $u_2 = 0$ |
| 3 | $z = h_1$ | $u_1 = u_2$ | $u_1 = u_2$ |
| 4 | $z = h_1$ | $k_{s1}\frac{\partial u_1}{\partial z} = k_{s2}\frac{\partial u_2}{\partial z}$ | $k_{s1}\frac{\partial u_1}{\partial z} = k_{s2}\frac{\partial u_2}{\partial z}$ |
| 5 | $t = 0$ | $u = p$ (a constant) | $u = p$ (a constant) |

and bottom, the two functions,

$$u_1 = \sum_1^\infty C_n \epsilon^{-TN''} \sin A_n \frac{z}{h_1} \sin \mu \nu A_n \quad \ldots\ldots\ldots\ldots (26a)$$

for the lower layer, and

$$u_2 = \sum_1^\infty C_n \epsilon^{-TN''} \sin A_n \sin \left(1 + \nu - \frac{z}{h_1}\right) \mu A_n \quad \ldots\ldots (26b)$$

for the upper layer, satisfy the respective differential equations and boundary conditions if $A_n$ is a root of the equation—

$$F(A) = \sigma \cos A \sin \mu \nu A + \sin A \cos \mu \nu A = 0 \quad \ldots\ldots\ldots (27)$$

and if

$$C_n = 2p \frac{\sigma \sin \mu \nu A_n + \sin A_n}{\sigma A_n \sin^2 \mu \nu A_n + \mu \nu A_n \sin^2 A_n} \quad \ldots\ldots\ldots\ldots (28)$$

In Eqs. 26, $N'' = (A_n)^2$ is used to simplify typography. The corresponding expressions for the average degree of consolidation in the lower and upper layers are

$$U_1 = 1 - 2\sum_1^\infty \frac{\sin \mu \nu A_n (\sigma \sin \mu \nu A_n + \sin A_n)}{(A_n)^2 (\sigma \sin^2 \mu \nu A_n + \mu \nu \sin^2 A_n)} (1 - \cos A_n) \epsilon^{-TN''} \ldots (29a)$$

and $U_2 =$

$$1 - \frac{2}{\mu \nu}\sum_1^\infty \frac{\sin A_n (\sigma \sin \mu \nu A_n + \sin A_n)}{(A_n)^2 (\sigma \sin^2 \mu \nu A_n + \mu \nu \sin^2 A_n)} (1 - \cos \mu \nu A_n) \epsilon^{-TN''} \ldots (29b)$$

For the case of drainage at the top only, the equations for hydrostatic excess pressure are:

$$u_1 = \sum_1^\infty C_n \epsilon^{-TN''} \cos A_n \frac{z}{h_1} \sin \mu \nu A_n \quad \ldots\ldots\ldots\ldots (30a)$$

and

$$u_2 = \sum_1^\infty C_n \epsilon^{-TN''} \cos A_n \sin \mu A_n \left(1 + \nu - \frac{z}{h_1}\right) \quad \ldots\ldots (30b)$$

in which $A$ must be a root of the equation—

$$G(A) = \sigma \sin A \sin \mu \nu A - \cos A \cos \mu \nu A = 0 \quad \ldots\ldots\ldots (31)$$

—and,

$$C_n = \frac{2\,p \cos A_n}{\sigma A_n \sin^2 \mu \nu A_n + \mu \nu A_n \cos^2 A_n} \quad \ldots\ldots\ldots\ldots (32)$$

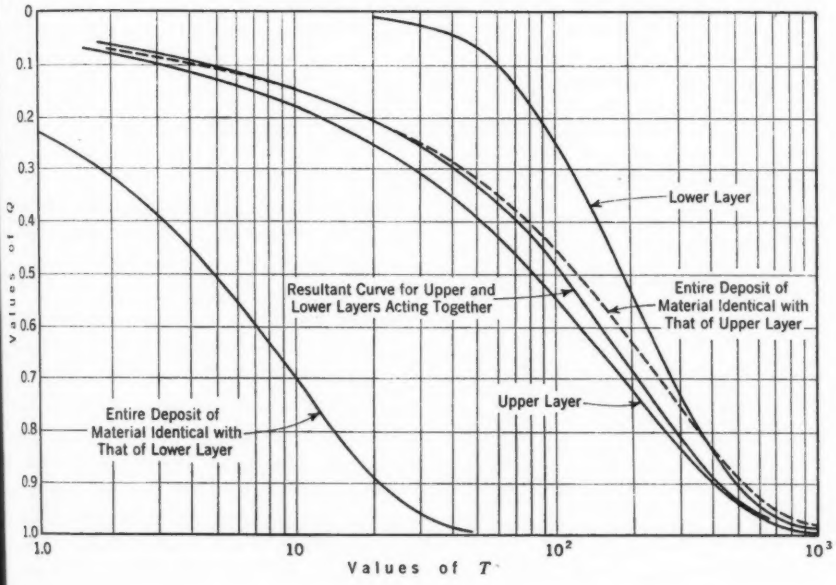


FIG. 6

The corresponding degrees of consolidation are:

$$U_1 = 1 - 2 \sum_1^\infty \frac{\sin A_n \cos A_n \sin \mu \nu A_n}{(A_n)^2 (\sigma \sin^2 \mu \nu A_n + \mu \nu \cos^2 A_n)} \epsilon^{-TN''} \quad \ldots (33a)$$

and

$$U_2 = 1 - \frac{2}{\mu \nu} \sum_1^\infty \frac{\cos^2 A_n (1 - \cos \mu \nu A_n)}{(A_n)^2 (\sigma \sin^2 \mu \nu A_n + \mu \nu \cos^2 A_n)} \epsilon^{-TN''} \quad \ldots (33b)$$

A convenient key to the application of these formulas is presented in Table 2. Results such as these are most readily comprehended if illustrated by graphical means, and the results of a particular case are shown in Fig. 6. In this situation it is assumed that there is drainage at the top only, with the following relationships between the coefficients and dimensions: $\nu = 4$; $e_1 = e_2$; $k_{s1}/k_{s2} = 10$; $a_1/a_2 = 0.4$; $\frac{c_1}{c_2} = \frac{k_{s1}}{k_{s2}} \frac{a_1}{a_2} = 25$; $\mu = \sqrt{\frac{c_1}{c_2}} = 5$; $\sigma = \frac{k_{s1}}{k_{s2}} \frac{1}{\mu} = 2$;

TABLE 2.—KEY TO DESIGN EQUATIONS

| Definition | Upper Layer | | Lower Layer | |
|---|---|---|---|---|
| | Drainage at top only | Drainage at top and bottom | Drainage at top only | Drainage at top and bottom |
| Hydrostatic excess pressure, $u$ | Eq. 30*b* | Eq. 26*b* | Eq. 30*a* | Eq. 26*a* |
| Degree of consolidation, $U$ | Eq. 33*b* | Eq. 29*b* | Eq. 33*a* | Eq. 29*a* |
| Coefficient, $C_n$ | Eq. 32 | Eq. 28 | Eq. 32 | Eq. 28 |

and $\mu\,\nu = 20$. Consequently, by Eq. 31,

$$G(A) = 2 \sin A \sin 20\,A - \cos A \cos 20\,A = 0 \quad \ldots\ldots\ldots (34)$$

The first eight roots of $G(A) = 0$ are given in Table 3.

TABLE 3.—ROOTS OF $G(A)$

| Root No. | Root Symbol | Root Magnitude | |
|---|---|---|---|
| | | Degrees | Radians |
| 1 | $A_1$ | 4° 5½′ | 0.0713 |
| 2 | $A_2$ | 12° 19½′ | 0.2150 |
| 3 | $A_3$ | 20° 39′ | 0.3604 |
| 4 | $A_4$ | 29° 06′ | 0.5079 |
| 5 | $A_5$ | 37° 39′ | 0.6571 |
| 6 | $A_6$ | 46° 16½′ | 0.8076 |
| 7 | $A_7$ | 54° 58′ | 0.9593 |
| 8 | $A_8$ | 63° 41½′ | 1.1117 |

Fig. 7 shows the distributions of excess fluid pressures, $u$, in the soil layers at two different times corresponding to $T = 100$ and $T = 200$. As is to be expected from the assumed relation between the soil coefficients, the lower layer consolidates as rapidly as the upper will permit, the variation in $u$ through the lower layer at any instant being very small. In the upper layer the variation is similar to that found in the case of a layer of homogeneous material with free drainage at one surface. In Fig. 6 the curves of $U$ versus $T$ are plotted for the upper and lower layers, together with a "resultant" curve computed from

$$U_r = \frac{U_1\,h_1 + U_2\,h_2}{h_1 + h_2} = \frac{4\,U_2 + U_1}{5} \quad \ldots\ldots\ldots\ldots (35)$$

Also shown are curves of $U$ versus $T$ which would result if the entire deposit of compressible soil were of material identical with that of (1) the upper layer, and (2) the lower layer.

Fig. 8 shows the "resultant" curve of Fig. 6, together with a curve obtained by averaging the coefficients of the two layers in the following manner, as suggested by Professor Terzaghi:[14]

$$a_{aver} = \frac{a_1\,h_1 + a_2\,h_2}{h_1 + h_2} = \frac{a_1\,h_1 + 2\tfrac{1}{2}\,a_1\,4\,h_1}{h_1 + 4\,h_1} = \frac{11}{5}\,a_1 = 2.2\,a_1 \quad \ldots\ldots (36a)$$

$$k_{aver} = \frac{h_1 + h_2}{\dfrac{h_1}{k_{s1}} + \dfrac{h_2}{k_{s2}}} = \frac{h_1 + 4\,h_1}{\dfrac{h_1}{k_{s1}} + \dfrac{4\,h_1}{k_{s1}}\,10} = \frac{5\,h_1}{41\,\dfrac{h_1}{k_{s1}}} = 0.122\,k_{s1} \quad \ldots\ldots (36b)$$

[14] "Sampling, Testing and Averaging," by K. Terzaghi, *Proceedings*, the Purdue Conference on Soil Mechanics and Its Applications, September 2 to 6, 1940, p. 151.

and

$$c_{aver} = \frac{k_{aver}}{\gamma\, a_{aver}} = \frac{0.122\, k_{s1}}{2.2\, a_1} = 0.0555\, \frac{k_{s1}}{a_1} = 0.0555\, c_1 \ldots\ldots (36c)$$

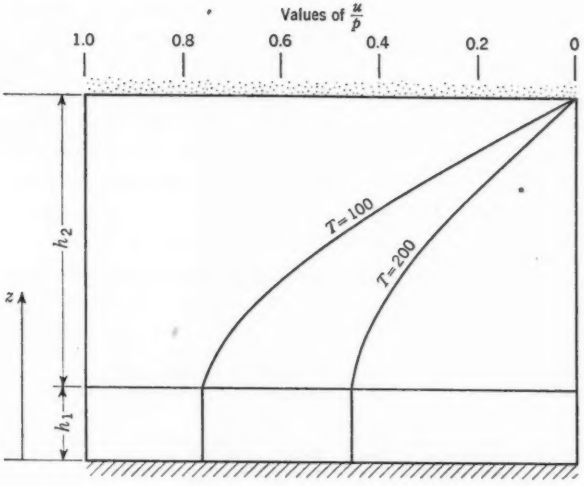


FIG. 7

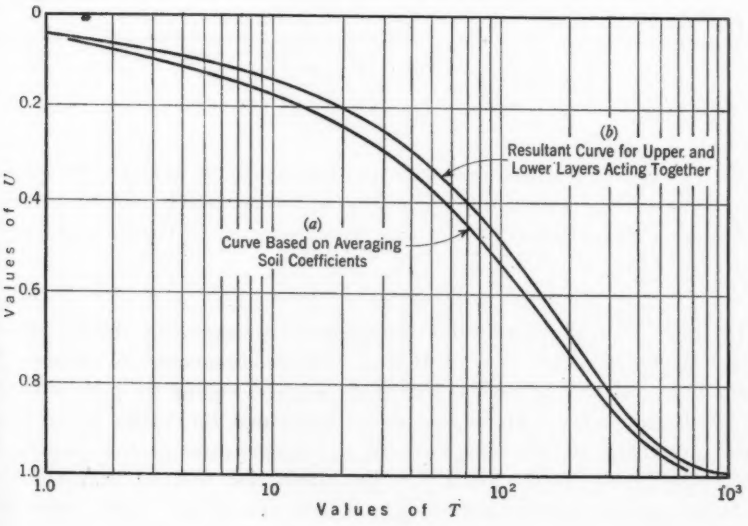


FIG. 8

Naturally, curve a, Fig. 8, secured by the method of averaging, is obtained with far less effort than the theoretically correct "resultant" curve which it approximates. It is evident from Fig. 8 that the discrepancy between the

two is not excessive. Fig. 9 shows these two curves plotted with $T$ on an arithmetic scale. The difference between the two at any given value of $T$ does not exceed 5% of the ultimate theoretical subsidence (that is, $U = 0.05$). The difference in $T$ for a given value of $U$ does not appear to exceed 20%. It is thought that the indicated agreement is adequate for practical purposes. Whether the agreement will be appreciably less close for other probable conditions can only be ascertained by "cut-and-try" methods. Such trials are time consuming, but, once values have been established, the degree of accuracy of the simpler method can be determined. This degree of accuracy need not be large, inasmuch as other variables, which enter the problem from sampling operation to estimates of the increase in load on the soil, all tend to influence the results in a more or less erratic manner.

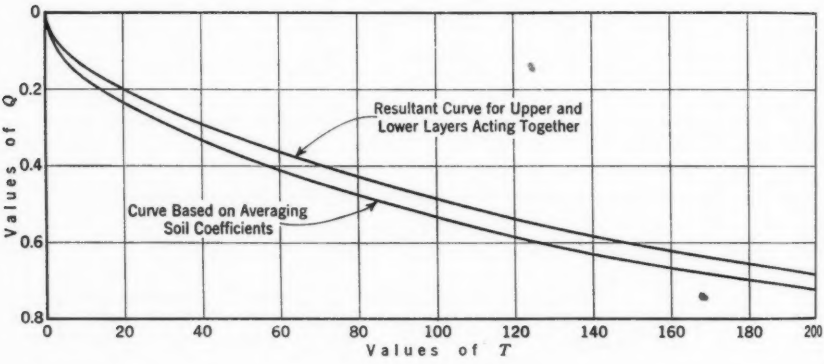


FIG. 9

For the case just considered, it is interesting to note that the theoretically correct curve lies between the curve secured by averaging, and that obtained by assuming the entire deposit to have the properties of the upper layer.

## ACKNOWLEDGMENT

In times past the writer has had numerous interesting discussions with George W. Glick of the firm of Moran, Proctor, Freeman and Mueser, New York, N. Y., relative to various technical subjects, among them the consolidation of clay deposits. These discussions prompted the writer to investigate matters further and led ultimately to the preparation of this paper. The writer is indebted to Mr. Glick for the stimulation derived from these conversations and for pointing out the existence of the solution[10] used in analyzing the influence of the incompressible drainage medium. This solution and a part of that pertaining to two consolidating clay beds was developed while the writer was in the employ of the firm, Moran, Proctor, Freeman and Mueser, which has consented to the use of Figs. 2 and 3.

# APPENDIX

## NOTATION

The letter symbols used in this paper conform, in so far as possible, with those listed in "Soil Mechanics Nomenclature." [15] The units of the quantity represented by each symbol are given herein, in terms of force ($F$), length ($L$), and time ($T$). Subscripts 1 and 2 are used to indicate when a symbol refers to soil layers designated respectively by I and II.

| Symbol | Definition | Unit |
|---|---|---|
| $A$ | = dimensionless number in Eqs. 27 and 31: $A_n$ denotes a root of Eqs. 27 and 31; | .... |
| $a$ | = coefficient of compressibility of soil; | $L^2\,F^{-1}$ |
| $C_n$ | = dimensionless coefficient defined by Eqs. 28 and 32; | .... |
| $c$ | = coefficient of consolidation of soil; | $L^2\,T^{-1}$ |
| $e$ | = void ratio of soil; | .... |
| $F$ | = "function of"; | .... |
| $G$ | = "function of"; | .... |
| $H$ | = thickness of soil layer: $H_d$ = thickness of drainage layer; $H_s$ = reduced thickness as defined by Eq. 1$a$; | $L$ |
| $h$ | = a vertical dimension in Fig. 1($a$); | $L$ |
| $k$ | = coefficient of permeability of soil: $k_a$ = "apparent" coefficient; $k_d$ = coefficient for the drainage layer; $k_s$ = reduced coefficient (Eq. 6); | $L\,T^{-1}$ |
| $M$ | = a substitution constant = $\frac{H_s}{H_d}k_d$ (see Fig. 3); | $L\,T^{-1}$ |
| $m$ | = an integer indicating a succession of values (Eq. 14) beginning with $m = 0$; | .... |
| $N$ | = a substitute exponent (see Eq. 8$b$), equal to $(r_n)^2$, used in formulas to simplify typography; | .... |
| $N'$ | = a substitute exponent (see Eq. 14) used in formulas to simplify typography; | .... |
| $N''$ | = a substitute exponent (see Eq. 26), equal to $(A_n)^2$, used in formulas to simplify typography; | .... |
| $n$ | = an integer indicating a succession of values (Eq. 9); | .... |
| $p$ | = pressure intensity; | $F\,L^{-2}$ |
| $R$ | = a ratio (Eqs. 8) = $\frac{H_s}{H_d}\frac{k_d}{k_s}$ | .... |
| $r$ | = a ratio: $r_n$ denotes a root of Eqs. 8; | .... |
| $T$ | = time factor: $T_a$ = a correct value of $T$ to determine a given value of $k_s$; | .... |

[15] *Manual of Engineering Practice No. 22*, Am. Soc. C. E., 1941.

| Symbol | Definition | Unit |
|---|---|---|
| $t$ | = elapsed time: $t_a$ = "apparent" time; $t_9$, etc. = time corresponding to $U = 0.9$, etc.; | $T$ |
| $U$ | = average degree of consolidation of a stratum: $U_p$ = degree of consolidation at a point; | .... |
| $u$ | = hydrostatic excess pressure in soil: $u_d$ = hydrostatic excess pressure in the drainage layer; | $F\,L^{-2}$ |
| $u_z$ | = hydrostatic pressure at point $z$, Fig. 1 and Eq. 2; | |
| $z$ | = a reduced dimension (Fig. 1); | $L$ |
| $\gamma$ | = density of the liquid (the definition of density $F\,L^{-3}$ differs from the one commonly employed. It is used here to avoid the introduction of the gravitational constant); | $F\,L^{-3}$ |
| $\epsilon$ | = base of natural logarithms; | .... |
| $\mu$ | = the ratio $\sqrt{\frac{c_1}{c_2}}$ (Eqs. 24); | .... |
| $\sigma$ | = a dimensionless ratio (Eqs. 24); | .... |
| $\nu$ | = a dimensionless ratio (Eqs. 24). | .... |

AMERICAN SOCIETY OF CIVIL ENGINEERS

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REPORTS

# FLOODS

## PROGRESS REPORT OF THE CHAIRMAN JOINT DIVISION COMMITTEE ON FLOODS

### Introduction

The exigencies of the war have prevented calling a meeting of the Joint Division Committee during 1943. This report was prepared by the Chairman to assist in visualizing the present-day status of floods and flood control in the United States. Summarizing, much has been accomplished but much remains to be done, for the annual bill for flood damage appears to roll up undiminished. On the one hand, we note the definitely salutary effects of flood-control works completed or in advanced stages of completion, and, on the other hand, a stupendous toll of damage. The conclusion must be that flood-control projects for the more densely populated areas must not be permitted to have their completion delayed. Every year of delay adds to the values invested in industries, homes, and lines of communication of all kinds—let us not forget the Big Inch oil pipe line washed out by the Arkansas River flood of May, 1943—thereby adding justification upon justification for the flood-control expenditures involved. The attention of engineers is becoming concentrated on the problems and technique of operating the rapidly increasing number of reservoirs provided with storage capacity for flood waters. These operation problems promise to occupy the center of the stage before long; and along with them comes a pressing demand for reliable long-range weather forecasting, primarily the meteorologist's job, but one in which the engineer can become proficient too.

### 1943—A Year of Many Floods

The year was ushered in by the worst flood in the upper Ohio River basin since the memorable St. Patrick's day high water of 1936. Water went surging through the "Golden Triangle" of Pittsburgh, Pa., and caused thousands to flee from their homes. Fortunately, effective use was made of such flood-control reservoirs as had been completed or were half completed, and this together with good flood-forecasting service caused Pittsburgh for once to celebrate coming through the ordeal better prepared and with much less loss

Note.—Please forward all comments on this Report directly to Chairman Gerard H. Matthes, Director, U. S. Waterways Experiment Station, Box 631, Vicksburg, Miss. Progress Reports are published in *Proceedings* only.

than formerly. Damage to industries engaged in war production assumed no startling figures, losses taking the form mostly of wasted working hours.

Not so happy was the news from Portland, Ore., also stricken during the first days of January by a disastrous flood of the Willamette River. March brought floods to Louisiana and southeastern Mississippi streams, and also to North Dakota; but these seemed almost local compared with the widespread floods which in April visited the vast section of the Middle West comprising Nebraska, Iowa, and Missouri and which were followed in May and June by great floods in the middle Mississippi River and tributaries. At St. Louis, Mo., the flood crest nearly equaled the record set in 1844. The inundations were particularly extensive in Illinois, Indiana, Missouri, Arkansas, Kansas, and Oklahoma. At one time as many as 154,000 people were homeless, thousands of United States troops were called on to render aid, and crop losses were faced on 2,756,000 acres of some of the most fertile lands in that area. July followed with floods in Minnesota, Wisconsin, and Iowa, thus completing one of the worst flood visitations that has every befallen the Middle West. Although the upper Mississippi River and all its tributaries were very high, no dangerous stages occurred in the lower river below Cairo, Ill. However, the high-water season was a protracted one and the average annual discharge for the water year ending September 30, 1943, was more than 690,000 cu ft per sec—well in excess of the long-term average. However, the St. Lawrence River, representing the outflow from the Great Lakes, reached a peak discharge of about 295,700 cu ft per sec, its highest in fourteen years. Summer and fall cloudburst floods exacted their usual annual toll. Steubenville, Ohio, and Belfast, Me., are examples of cities that suffered disruption of their municipal water supplies through flash floods.

### Work for State Planning and City Zoning Commissions

Important factors in maintaining flood damage at high levels appear to be the increase and shift in population, and the ever-present tendency to locate industrial establishments and dwellings on level land in river bottoms or on alluvial plains—land which owes its very existence to flood deposits. Lack of foresight and high-pressure salesmanship often account for much property being thus incorrectly located in the paths of floods. The Village of Spring Valley, Wis., is considering moving to higher ground. Located in the valley of the Eau Galle River, it suffered six floods within one year (1942). Each inundated the town and the last one wrecked its water works. Thus, its citizens were convinced that it would be cheaper to move the town than to continue to rebuild it. Perhaps, Spring Valley should consider itself lucky. Had the six floods come ten years apart the village might have grown into a city before its flood hazard would have been clearly visualized and another flood-ridden community would have been added to an already long list.

State planning and city zoning commissions should take heed: Once cities grow to be large and their populations dense, the cost of zoning them tends to become prohibitive. London (England) on the Thames, Paris (France) on the Seine, Cologne (Germany) on the Rhine, Breslau (Germany) on the Oder, and Rome (Italy) on the Tiber are great cities which have struggled for centuries

with their flood troubles without conquering them. As recently as February, 1937, Cologne suffered considerable damage to the low part of the city by a flood of the Rhine. Breslau, a city of more than 600,000, is built on both banks of the Oder River whose watershed is equipped with twenty-one flood-control reservoirs and retarding basins. On September 3, 1938, the greatest flood in the record of the Oder River inflicted great losses throughout the Oder Valley, despite numerous local flood-protection works, and caused extensive damage at Breslau. For the protection of Breslau alone more than twice as much reservoir capacity should have been provided—something which apparently is not within economic reach.

Rome considered flood control as far back as the days of Tacitus who describes the ravages by the Tiber River and the appointment of a commission of Roman senators to formulate a flood-control plan. The answer appears to have been that the cost was not then economically justified, because many centuries elapsed before flood walls and dikes were built for the protection of Rome. The flood of December 17, 1937, outmatched these protective works, and official reports placed the damage suffered in Rome alone at $4,000,000.

London is the outstanding example of a city which grew to such unwieldly size and became controlled by so many governing bodies that repeated attempts at providing flood control were frustrated until 1937, when a commission representing all municipal and governmental interests was appointed to deal with the situation in a comprehensive way. London's flood problem (which dates back to the year 1091 when the first London Bridge was swept away) is complicated by a combination of headwater runoff and high tides in the Thames River. Strange as it may seem, London's most precious possessions: The Parliament Buildings, the Tower of London, treasure-laden Tate Art Museum, Westminster Abbey, the grounds of Windsor Castle, Eton College, and many others are located within the flood zone. The long record of flood devastations makes impressive reading.

## "Flood-Free Johnstown"

Citizens of Johnstown, Pa., are eagerly announcing the early completion of an ambitious improvement of the Stony Creek and Conemaugh River channels designed to provide ample capacity for flood discharge, which will "free Johnstown" for all time of its flood menace. Since May 31, 1889, the memorable date when South Fork Dam broke and caused a loss of 2,300 lives in this community, which then had a population of but 20,000, storm flood waters have repeatedly invaded the city. In fact, less than two years later (February 17, 1891) Johnstown was inundated by a flood—that time without any dam failure to obscure the true nature of its flood problem. The climax came in 1936 when one third of the city, which by then had a population of 70,000, was inundated, inflicting property losses amounting to more than $40,000,000, or about three times the monetary losses sustained in 1889. Model experiments in 1938–1939 demonstrated the practicability of a channel-rectification project which has since been completed under the direction of the U. S. District Engineer in Pittsburgh, at a cost of nearly $9,000,000.

### Highway Damage Leads

The huge network of highways and roads in the United States is today the chief sufferer in terms of dollars and cents; like the railroads, washouts by cloudbursts remain the unsolved problem. However, although bridge and culvert losses are not caused solely by cloudburst runoff, they continue to mount into very large figures. The State of Pennsylvania in 1942 counted 167 bridges damaged and 69 destroyed in a period of 128 days by ten summer floods. The total damage to highways exceeded $2,340,000, of which about $1,000,000 was to bridges. A program of channel clearing and improvement was authorized by the governor. The State of New Mexico during the record floods of May and September, 1941, in the Pecos, Cimarron, and Canadian river valleys sustained highway and bridge losses placed at more than $8,000,000—more than the entire highway budget for that year. The disastrous storm of September, 1938, in New England did property damage that has been difficult to evaluate but appears to have been in the neighborhood of $400,000,000. Damage to highways and especially bridges ran into large figures, and repairs have extended over a long period of time. During the great floods of March, 1938, in Southern California, damage to highways and bridges was also the largest single item, amounting to $13,774,000 or nearly twice the damage to valuable orchards and high priced crops, and more than twice the residential damage.

In occasional instances, bridges which had been rebuilt were damaged again by later floods, often due to undermining of abutments or piers. In this connection, a vast amount of experimental work in the past has been aimed at discovering the effect of bridge-pier shapes on stream flow, but apparently little has been done to determine the extent to which various pier shapes affect scour of the river bed. This is a field worthy of investigation, in which careful field determinations are needed in conjunction with model studies. Of interest is the rule of thumb, derived from hard experience, used by railroad engineers in the Great Plains region of the United States, where river beds are notoriously sandy and subject to deep scour during floods. This so-called 4 to 1 rule requires that bridge foundations be placed to allow for a bed scour of 4 ft in depth for each foot of rise in water surface above the low-water plane.

### Dikes and Levees

In the past dike and levee construction in the United States has been taken much for granted. The steadily increasing height of these embankments for the protection of valuable properties, such as industries engaged in war production and airports, is calling for greater care in construction and maintenance. Greatly increased flood stages have resulted from more co ıplete confinement of flood flow and are producing greater hydrostatic heads on levee foundation materials, thereby promoting percolation through the latter. In some areas lands presumed to be safely protected have become subject during high river stages either to general seepage or to localized boils. Both are damaging to agriculture, as well as to industrial and municipal interests, and are beginning to call for remedial measures, often entailing large expenditures. Seepage, if left uncontrolled, has been responsible in many instances for levee instability

and occasional failures. Experiments have been and are being made with pressure-relief well installations and with surface drainage by ditching. The addition of a berm on the land-side toe of the embankment still is considered good practice despite its great cost. Summarized, the lowly dike or levee of old is making way for massive modern earth embankments tending to assume the attributes of dams. These are engaging the attention of soil mechanics experts, hydraulic engineers, and geologists.

The problem of what is the safe height of a levee or dike is no longer the moot question which it once was. Physically, the limit of height most often is imposed by the safe bearing value of the soil on which the levee must be built, whenever seepage tends to become excessive. In addition, there is a psychological limit which must not be ignored. People enjoy the safety provided by a well-built levee, but they do not enjoy living behind a structure so high that they become afraid of it, or have reason to doubt its stability on other counts. Fear causes people to sell and move away, causes property values to depreciate, and serves to defeat the very objectives which flood control is meant to serve.

## Reservoir Operation

Reservoir operation looms as the momentous problem for the future. War Department engineers have been devoting much study to the complexities of operating the rapidly increasing number of reservoirs for flood control combined with other water uses, or for flood control alone. These studies, of course, are aimed at utilizing available storage capacities as effectively as possible, but are concerned more particularly with the release of the impounded waters. If left to uncoordinated local action, the emptying of groups of reservoirs may be full of grave dangers. To assist in visualizing these problems in a practical way, a gigantic model of the 1,240,000-sq mile Mississippi River watershed is being built, in which will be reproduced at a horizontal scale of 1 : 2,000 all principal flood arteries, all existing and proposed reservoirs, and all levee systems. Originally conceived by Maj.-Gen. Eugene Reybold, M. Am. Soc. C. E., Chief of Engineers, U. S. Army, this model, for which ground was broken near Jackson, Miss., in July, 1943, will cover the better part of a square mile of land surface. The work so far has consisted of grading the terrain with the aid of some two thousand prisoners of war working under the direction of engineers from the U. S. Waterways Experiment Station, Vicksburg, Miss.

## Flood Control for Private Interests

Although the public mind has become trained to regard flood control as a governmental institution, the fact remains that there are still flood-control jobs to be done for private and semiprivate interests which call for consulting service. Projects of this kind are on a less pretentious scale, and, since they call for expenditures by individuals or business interests, the economic aspect is different. Thus, whereas in government projects protection against maximum possible floods has been made a basic consideration, or at least serves as a starting point in planning, the private client as a rule must content himself with seeking partial protection. The engineer's problem in such cases is one of providing only as much protection as the client's interests demand. The

client most often will look upon flood protection in the same light as he does fire or life insurance. He buys no more than he can afford. What then is a prudent investment? This is the question which the engineer must assist in answering, and in this problem the all-time maximum possible flood becomes a purely hypothetical event which may evoke no practical interest. In problems of this kind, however, the engineer of today has at his command an array of flood data which his predecessor, a quarter of a century ago, did not have and which enables him to give advice in the matter of probable frequencies of recurrence of floods of various magnitudes and from different causes.

### Beneficial Use of Flood Waters

The "Upstream Engineering Conference" held in Washington, D. C., September 21–24, 1936, in which the pros and cons of arresting storm runoff at the source by using ponds and by promoting infiltration through the medium of reforestation and soil stabilization were discussed, proved helpful in showing that no one method of water conservation or of flood control offers a panacea. In the humid regions of the United States, headwater conservation and downstream flood control and flood-water utilization may perhaps go hand in hand. In the western arid regions, where water is at a premium and virtually the essence of life, flood waters originating at the very heads of rivers have a cash value for irrigation and domestic uses, and any plan which promotes water losses by distribution into the soil at the higher levels, where most of the precipitation occurs, becomes inimical to man's existence. The divergence in thought which has existed between East and West, as regards making beneficial use of waters including flood flow, is wearing off. Water has become scarce in many eastern areas. New Jersey, whose State Water Policy Commission finds industrial expansion hamstrung for lack of water supply, represents an outstanding example.[1] Even in so wet an area as the lower Mississippi Valley water shortages have developed in the rice growing districts. Many of our western states abrogated the antiquated law of riparian rights years ago in order to make beneficial utilization of water practicable under the law of appropriation, which in substance states that beneficial use shall be the measure and the limit of the water-user's right. The time seems not far distant when states of the humid region may have to begin thinking in like terms, as their water supplies become progressively scarcer. Merely evacuating flood waters to the sea, as by the retarding-basin principle for instance, may serve to control floods but effects no conservation for beneficial use.

### Flood Data

In its first annual report dated December 26, 1934,[2] the Committee on Flood Protection Data stated:

> "The greatest good to the user of flood data will result from placing before him just as complete an array of flood events as historical and engineering records can be made to yield. This relieves him of the necessity of

[1] Memorandum of the New Jersey State Water Policy Commission to the Congressional Committee on Rivers and Harbors in the matter of H.R. 1880, 78th Cong., 1st Session, authorizing the Construction of the New York Bay—Delaware River Section of the Atlantic Intracoastal Waterway: Dated March 8 1943.

[2] *Proceedings*, Am. Soc. C. E., March, 1935, p. 336.

undertaking research work of this kind at a time when his energies usually are fully occupied."

Little did the committee then dream that within ten years the profession would find itself equipped with a wealth of data, published in admirably usable form. Engineers and meteorologists have collaborated in this gratifying result. Notable features of many of the later reports are the attention paid to crest stages at points up and down the streams, the publication of storm rainfall data, and also the publication of sufficient stage and discharge figures to permit plotting flood hydrographs. Special mention is due the excellent information available on the earlier flood occurrences. Authenticated heights attained by great floods of a century or longer ago have been helpful in dispelling the traditional belief that such floods are primarily attributable to deforestation, overgrazing, and the clogging of stream channels with erosional debris. These contributory factors for a while tended to become greatly overemphasized. We know now that great floods, which would have been of disastrous proportions were they to happen today, occurred in: 1692 on the Delaware River at Trenton, N. J.; 1763 on the Ohio River at Pittsburgh; 1770 on the Los Angeles River; 1771 on the James River near Richmond, Va.; 1785 on the Connecticut River in Massachusetts and also on the Androscoggin and Kennebec rivers of Maine; 1793 on the Cumberland River at Nashville, Tenn.; 1828 on the Rio Grande at San Marcial, N. Mex., and El Paso, Tex.; 1844 on the Mississippi River at St. Louis; 1850 and 1852 on the Sacramento River at Sacramento, Calif. The unusual research which led to the unearthing of some of these early happenings would, if assembled, read like an epic in engineering research. Such sources as the diaries of early settlers, the baptismal record of an old Spanish mission, and the files of historical societies and those of the British Museum were investigated. Most of these early floods occurred before man's ruthless hand had made appreciable inroads into the forests or had promoted soil erosion. Yet, more than a century ago the belief already prevailed that timber cutting was responsible for the flood disasters of that time,[3] while the equally unwarranted belief that the winters are becoming less cold had been expressed as far back as 1786.[4] It seems impossible to eradicate such popular notions.

## Inundations Caused by Ice

A review of overflows caused by river ice reveals that startlingly little headway has been made in this country in combating ice gorging. The graphic accounts of privations and damage suffered because of ice floods in such rivers as the Hudson, the Delaware, the Susquehanna, and the Potomac in 1904, 1918, and as late as 1936 bear an unescapable similarity to the equally graphic accounts of such damage in 1861, 1846, 1816, and 1784. In only two respects has progress been made; namely, in providing more substantial bridges, dams, and other structures along rivers, and in the issue of more timely warnings. The latter is doing much to save lives and lessen property damage. Ironically, as if to mock man's lack of ingenuity, some of our rivers which once were no-

[3] *Hazard's Register of Pennsylvania*, 1828.

[4] *Columbian Magazine*, November, 1786.

torious for ice gorging have become practically free from ice as an incidental result of their waters being used for condensing purposes by industries. One reason why flood-control engineers have paid little attention to ice gorging is because it is primarily a low-water phenomenon. Yet, on some rivers affected by ice, the highest overflow stages on record are attributable to ice jams. The logical way to attack this neglected phase of flood control would seem to be the collection of factual data and the publication of a chronological record of the ice troubles for each of the principal rivers affected, together with estimates of the damage which resulted.

## Recent Advances in Flood-Control Engineering

Signal progress is being made in several directions. Although it may be somewhat early to make specific comment, all signs point toward less mathematical pondering over the probable frequencies of recurrence of great floods and less effort being devoted to finding new ways of "routing floods" through reservoirs and down river channels. Instead, a steadily increasing trend is developing toward intensely practical observations, either at first hand or with the aid of large models, on the behavior of flood flow, valley storage, time of travel of flood water in channels and in reservoirs, and on the problems confronting reservoir operators. Aspects which thus far have remained moot subjects, calling for assumptions, coefficients, and other factors of ignorance, are undergoing dissection and are becoming better understood. In this endeavor, two organizations in particular, the Tennessee Valley Authority (TVA) and the Mississippi River Commission, both well equipped for the task, have made noteworthy contributions during the past year. Many United States Engineer Offices are likewise studying their operation problems in a practical way. Hydrologic data are being collected at more stations than ever before, and their analysis is entering the hitherto untrodden field of establishing relations between watershed types and rates of runoff. River channel morphology and the transportation of sediment are being subjected to new forms of research and some old established misconceptions are going into the discard, particularly as regards bar building and scour in natural channels. Only a few of the more important contributions of the year can be mentioned:

(*a*) *Hydraulics of Culverts.*—Much has been accomplished by state highway departments on the improvement and standardization of culvert design. A variety of local conditions imposed by the slope of the terrain and runoff characteristics is being considered. Formerly these conditions were not accorded the importance due them. Noteworthy in this connection are the analytical studies in culvert hydraulics made under the direction of the State Highway Engineer of California, and their publication during 1943 in serial form.[5] These articles collectively amount to a manual on culvert practice which should be helpful to highway engineers generally. Important contributions in their preparation were made by R. Robinson Rowe, M. Am. Soc. C. E.

(*b*) *Flood Forecasting.*—Flood forecasting from a model was attempted for the first time during the floods of 1943 by engineers of the Mississippi River Commission. The model comprised stretches of the Ohio and Middle Missis-

[5] *California Highways and Public Works*, Sacramento, Calif., 1943.

sippi rivers and their confluence at Cairo where they form the Lower Mississippi River. Included in the model, which covers more than an acre of ground, are parts of the Cumberland and Tennessee rivers, the prehistoric Ohio River (Cache) Valley which connects the two main rivers north of Cairo, and the entire Birds Point-New Madrid Floodway opposite Cairo. By simulating in the model actually recorded upriver stages received by telephone, the model quickly indicated the stages to be expected at points downstream. The results, as checked against subsequently observed stages, have demonstrated the practicability of this method as being peculiarly adapted to river situations at and below confluences of major tributaries. The same model was utilized also for determining the volumes of water held in temporary overbank or "valley" storage for different flood stages, by the rather unique process of partitioning off the model into 10-mile compartments and calibrating these by filling with water. Accurate knowledge of valley storage quantities is essential in any flood-forecasting procedure using discharge data. Its determination, however, is not as a rule readily accomplished from maps or by computation from flood-flow data.

(*c*) *Translatory Waves.*—Waves on the Clinch and Tennessee rivers were studied to discover the speed with which they traveled downstream. The observations were made by releasing carefully determined volumes of water from Norris Dam at times when tributary inflow was negligible. The 74-mile-long Wheeler Reservoir was also used for observations of this kind. The observed translatory wave movement was compared with the computed results using formulas which have long since needed checking. The studies were made by J. H. Wilkinson,[6] M. Am. Soc. C. E., under the general direction of Sherman M. Woodward, Hon. M. Am. Soc. C. E., consultant for the TVA.

(*d*) *Geology as an Aid in Flood Control.*—Among the more remarkable products of the year is the completion of a geological investigation in the huge alluvial valley of the Lower Mississippi River, the subsurface composition of which has long been an enigma to engineers concerned with flood control, river regulation, and drainage. Lack of information had become the cause of widespread seepage conditions, levee instability, and difficulties in stabilizing the navigable channel of the river. Thirty months of field work, the examination of some 17,500 logs of borings, and thousands of aerial photographs were required in order to trace the old channel locations of the prehistoric rivers which, since the last glacial epoch, have traversed the valley. Clay beds, sands, gravels, and silts found their appointed places in this scheme of things, and now the why and wherefore of their respective occurrences and their bearing on levee foundations and river maintenance problems are for the first time understood. The study was made by Harold N. Fisk, professor at the School of Geology, Louisiana State University, at Baton Rouge, under the direction of Brig.-Gen. Max C. Tyler, M. Am. Soc. C. E., who conceived the plan for the investigation.

(*e*) *River-Bed Retrogression.*—The annual report of the Bureau of Reclamation on the rate of river-bed retrogression in the Colorado River below Boulder Dam continues to develop data of unusual interest to engineers concerned with

[6] *Proceedings*, Am. Soc. C. E. (publication pending).

the construction of high dams on heavy sediment bearing streams. The complete wrecking of this large river channel due to the cessation of its natural regimen, and the deterioration of the great delta of the Colorado River which has also resulted, afford much food for thought. Although perhaps no intrinsic damage has resulted in this particular instance, similar conditions on rivers elsewhere might be attended by very serious consequences.

(*f*) *River-Discharge Regulation.*—Regulating the flow of the Tennessee River and its tributaries by means of some twenty multiple-purpose dams and reservoirs having a combined capacity of from 7,000,000 to 8,000,000 acre-ft reached a remarkable degree of perfection during 1943. The low-water flow of the Tennessee, in times past, has been known to drop as low as 5,000 cu ft per sec. In 1943, a year of considerable deficiency in rainfall, regulated flows in the Lower Tennessee River were maintained near 25,000 cu ft per sec throughout the low-water season. Two additional reservoirs, the Kentucky Reservoir near the mouth of the river and the Fontana Reservoir on the Little Tennessee River, with prospective storage capacities of 4,600,000 acre-ft and 1,200,000 acre-ft, respectively, are nearing completion and will make the Tennessee River and its tributaries the most perfectly regulated river system of its size in the world. Long-range weather forecasting is being developed as an important adjunct in these operations. Nicholls W. Bowden, M. Am. Soc. C. E., is head of the River Control Section of the TVA. Other large American rivers whose regulation has shown marked progress are the Sacramento and Colorado rivers.

(*g*) *Solving the Riddle of Stream Meander.*—New light has been shed during the year on the age-old controversial subject of the meandering habits of rivers. A series of systematically conducted observations extending over a year at the Waterways Experiment Station under the personal direction of General Tyler, President of the Mississippi River Commission, has been of considerable assistance in clearing away the confusion of ideas that has prevailed among engineers. The need for an exhaustive study had been felt for some time as the subject intimately concerns the flood-carrying capacity of a meandering stream and the manner in which it builds its bars and imposes limits on navigable depths. In turn, a meandering river is definitely affected by the confinement of its flood flow by levees and dikes, and this phase has been included in the experimental inquiry. In many of the tests, water flowing unrestrained was permitted to shape its own channel; in other tests the effects of revetments, spur dikes, channel shortenings, and vegetation were studied. A variety of basic conditions was introduced, such as combinations of slope, discharge, and granular materials of different grain size, specific gravity, and cohesion. In many respects, the experimental work goes well beyond the basic aspects of meandering. It covers the fundamentals of sediment transport by streams of all types, and promises to be helpful in eliminating old misconceptions regarding the mechanics of bar building and scour, bank caving, the building of new banks, and the reasons why channels aggrade, degrade, or remain "poised." Collectively, these studies which were conducted by Joseph F. Friedkin, Assoc. M. Am. Soc. C. E., Captain, Corps of Engineers, U. S. Army, and which are still in progress, already constitute a valuable contribution to the science of rivers

(potamology) as well as to the art and science of flood control. The results obtained to date have been an important factor in enabling the Mississippi River Commission to announce at recently held public hearings its ability to increase the prescribed 9-ft navigable depth at low water in the Mississippi River below the mouth of the Ohio River to 12 ft, down to deep water below Baton Rouge. The river, today, is generally considered to have the best channel it has ever had, and is an important carrier of war materials and war ships built at inland points. In addition, its flood-carrying capacity has become greatly increased.

### Geographical Distribution of Flood Disasters

In order to visualize what sections of the United States are most affected by floods, Table 1 was prepared from data published by the American Red

TABLE 1.—Number of Flood Disasters, in The United States, July 1, 1920, to June 30, 1940, From Data on the Disaster Relief Operations of the American Red Cross

| States | Number |
|---|---|
| *(a) Northeastern* | |
| Ohio | 20 |
| Pennsylvania | 19 |
| New York | 14 |
| Indiana | 13 |
| Massachusetts | 6 |
| Connecticut | 4 |
| New Hampshire | 4 |
| New Jersey | 3 |
| Maine | 2 |
| Vermont | 2 |
| Michigan | 1 |
| Rhode Island | 1 |
| Delaware | 1 |
| Total | 90 |
| *(b) Rocky Mountain and Interior Basin* | |
| Colorado | 10 |
| New Mexico | 9 |
| Wyoming | 6 |
| Montana | 5 |
| Utah | 4 |
| Idaho | 4 |
| Arizona | 4 |
| Nevada | 2 |
| Total | 44 |
| *(c) Pacific* | |
| California | 19 |
| Washington | 7 |
| Oregon | 5 |
| Alaska | 0 |
| Total | 31 |
| *(d) Midwestern* | |
| Arkansas | 21 |
| Oklahoma | 21 |
| Illinois | 20 |
| Missouri | 18 |
| Kansas | 18 |
| Iowa | 16 |
| Wisconsin | 11 |
| Nebraska | 9 |
| South Dakota | 4 |
| North Dakota | 2 |
| Minnesota | 1 |
| Total | 141 |
| *(e) Southeastern* | |
| Kentucky | 34 |
| West Virginia | 17 |
| Tennessee | 14 |
| Georgia | 13 |
| South Carolina | 7 |
| Virginia | 6 |
| Maryland | 5 |
| North Carolina | 3 |
| District of Columbia | 2 |
| Total | 101 |
| *(f) Gulf Coast* | |
| Texas | 35 |
| Mississippi | 21 |
| Louisiana | 13 |
| Alabama | 11 |
| Florida | 9 |
| Total | 89 |

Cross relating to disaster relief operations. As the period covered is but 20 years (July 1, 1920, to June 30, 1940) and only 496 flood disasters required

relief, the data are not conclusive. They may however be accepted as indicative. In interpreting this table the size of any state will be found to be a factor but not as important as density of population. Texas, the largest state, is traversed by many large river systems and also is subject to the greatest storm rainfall. Thus, it necessarily heads the list. In general, Table 1 indicates that there are large areas of the United States subject to frequent visitations where as yet relatively few flood-control works have been built.

Gerard H. Matthes, *Chairman*
*Joint Division Committee on Floods*

December 31, 1943

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# CONCRETE RESERVOIRS OF THE VERTICAL-BEAM TYPE

**Discussion**

BY C. MAXWELL STANLEY, JR.

C. MAXWELL STANLEY, JR.,[10] M. AM. SOC. C. E.[10a]—Some of the discussers favor the vertical-beam type of construction for concrete reservoirs and some question the value of this design. Both comments are needed for a proper evaluation of any new method of design.

The examples of vertical-beam type construction given by Messrs. Cochrane and Pérez Guerra apply to conditions somewhat different from those encountered in the reservoirs described by the writer. The reservoir designed by Mr. Cochrane is of particular interest: It illustrates the application of the method to a circular reservoir of large diameter as suggested in the paper; and it extends the method to a greater degree of backfilling than the writer has considered desirable. As Mr. Cochrane states, the internal pressure normally will exceed the external pressure and cracks that may occur in the inner face of the wall when the tank is empty will tend to close when the reservoir is filled above the normal drawdown level. The arch wall design described by Mr. Cochrane is a proper addition to the retaining wall type listed under the heading of "Conventional Design: Retaining Wall Types."

Messrs. Schmidt and Koon and Gearhart comment on the desirable percentage of horizontal reinforcement.[10b] The writer agrees that the amount of horizontal reinforcement in projects No. 1 and No. 2 is less than that desirable. Two reservoirs designed and constructed since the preparation of the paper (December, 1942) have had approximately 0.4% horizontal reinforcement which is comparable to that of project No. 3. No indication of cracking appeared on either of these reservoirs as late as fourteen months after construction.

The discussion of Messrs. Koon and Gearhart is an able presentation of the

NOTE.—This paper by C. Maxwell Stanley, Jr., M. Am. Soc. C. E., was published in December, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1943, by Lewis A. Schmidt, Jr., M. Am. Soc. C. E.; April, 1943, by Messrs. Victor H. Cochrane, and R. E. Koon and J. C. Gearhart; and May, 1943, by Gustavo Pérez Guerra.

[10] Cons. Engr. (Stanley Engineering Company), Muscatine, Iowa.

[10a] Received by the Secretary December 20, 1943.

[10b] In March, 1943, *Proceedings*, p. 456, line 6 of the text, change 0.11% to 0.17%.

objections that may be raised against the vertical-beam design. Comments on these objections are in order. There are many factors that affect the selection of the type of reservoir, and, where those factors are such that the vertical-beam type of wall cannot be applied, obviously some other design must be used. Although the vertical-beam design is not universally applicable, it has certain advantageous features that may be utilized where it is applicable. The vertical-beam design need not be eliminated from consideration because a part of the water capacity is below natural grade, for in such cases a design similar to that shown in Fig. 8 may be used with the bottom of the wall supported by

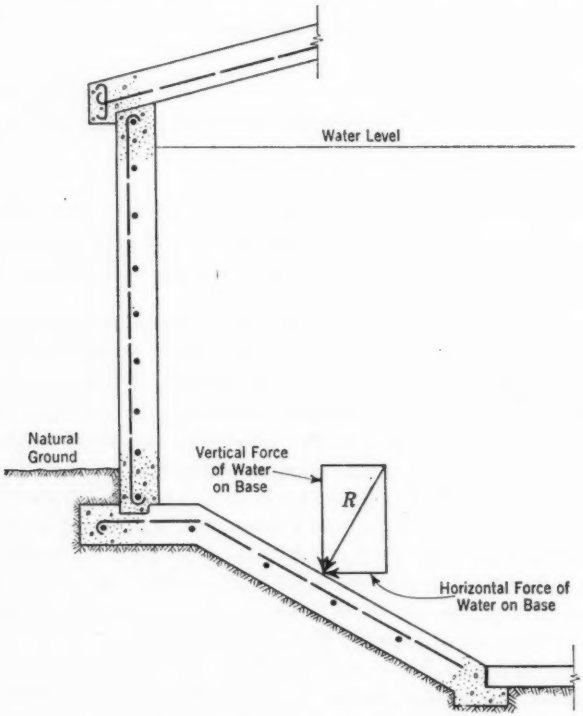


FIG. 8.—APPLICATION OF VERTICAL-BEAM DESIGN TO RESERVOIR WITH PART OF STORAGE BELOW NATURAL GRADE

the frictional resistance resulting from the weight of the water on the base. This construction is only a modification of the method of bottom wall support shown in the lower example of Fig. 3(*b*). The vertical-beam design is not applicable to circular reservoirs, except those of relatively large diameter, but seldom is a circular reservoir required by local conditions. Rather the circular reservoir is more often the result of the type of design selected. There are few instances in which a square or rectangular shape cannot be used if desired. In general, the vertical-beam design may be applied where any of the "conventional designs" are considered, provided the height of wall is not too great

for the wall to be poured in one lift. Moreover, local conditions as well as the ratio of depth and area will vary cost relationships between the various designs.

The hypothesis, presented by the writer, that a better design is obtained when the surfaces in contact with the water are in compression can probably be proved or disproved only by research which has not yet been undertaken. The writer believes thoroughly that having compression concrete in contact with the water is advantageous. This belief is based on his observation of numerous concrete structures and is confirmed by observed experiences with the Hewitt type of pre-stressed circular reservoir.

Cost comparisons are extremely difficult. Those made by Messrs. Koon and Gearhart assume the same unit price for concrete for the two types of construction—namely, project No. 1, Fig. 5(*a*) and the alternate Fig. 6(*a*); and, likewise, for project No. 3, Fig. 5(*c*) and the alternate Fig. 6(*b*). The unit prices for the two types of construction should not be the same. Moreover, the comparison made by Messrs. Koon and Gearhart in Fig. 6(*a*) assumes a design with only a 15-ft wall height for the cantilever type. If such an arrangement is acceptable for the cantilever type, it may also be used for the vertical-beam type, employing a construction similar to that shown in Fig. 8.

For these reasons the writer questions the cost comparison made by Messrs. Koon and Gearhart. It has been his experience, as indicated in the paper, that the cost of the vertical-beam reservoir compares favorably with conventional types. Table 4 shows the construction cost of several projects

TABLE 4.—COST DATA[a] ON VERTICAL-BEAM RESERVOIRS

| Project | Contract cost[b] | Date awarded | ENR cost index | Contract cost[c] | Capacity (gallons) | Adjusted cost[d] |
|---|---|---|---|---|---|---|
| 1 | $39,750 | August, 1935 | 93.82 | $54,800 | 3,000,000 | $18,300 |
| 2 | 4,375 | November, 1938 | 112.69 | 5,040 | 120,000 | 42,000 |
| 4 | 2,710 | March, 1942 | 129.65 | 2,710 | 60,000 | 45,200 |

[a] Costs do not include land, engineering, or inspection. [b] Cost of project No. 1 includes all piping for inlet, outlet, overflow, etc.; the cost of projects No. 2 and No. 4 does not include piping. [c] Adjusted to March, 1942. [d] Dollars per million gallons.

adjusted to January, 1942, prices using the *Engineering News-Record* (ENR) cost index. These costs seem self-explanatory.

If construction joint spacing of greater than 20 ft is adequate, then this should be used, and may be used with the same advantage for the vertical-beam type of construction as for any of the conventional types.

The writer agrees that the use of unprotected tie rods is ill-advised, and, on the projects described, adequate means have been taken to protect them. The methods have varied, but for tie rods buried under the floor the following procedure is suggested: (1) Design the bars over size to allow for some corrosion; (2) give the bars two or more coats of good asphaltic waterproofing material and wrap them in burlap, sisal-craft, or similar material to protect the waterproofing; and (3) place and backfill the bars carefully with sand (for protection and drainage) so as to shield them by the slab placed above. These

means of protection are not dissimilar to those used on steel pipe and if carefully applied and inspected will give adequate protection. However, if one is skeptical of the use of tie rods, they may be eliminated and the vertical-beam design may still be used by placing reinforcement in the top slab (middle illustration in Fig. 3(*a*)) to support the top of the wall and by using the weight of the water to resist the horizontal thrust at the bottom of the wall (bottom illustration in Fig. 3(*b*)).

Objection to the use of surface waterproofing on the inner walls of reservoirs is largely a matter of opinion; it applies equally to all types of design and need not be argued.

In conclusion the writer wishes to reaffirm his statement that the vertical-beam method of reservoir construction has been developed and tested so that its advantages and disadvantages can be established. In those instances where conditions permit its use, it has certain advantages and affords a simple and economical type of construction.

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DISCUSSIONS

# PRIMARY RÔLE OF METEOROLOGY IN FLOOD FLOW ESTIMATING

## Discussion

BY MERRILL BERNARD

MERRILL BERNARD,[13] M. AM. SOC. C. E.[13a]—Mr. Gilman raises the natural doubt that the maximum storm necessarily produces the maximum flood. The writer's definition of the ultimate or maximum storm will be found to qualify magnitude in considering the principal exterior influences on floods—the snow-melt contribution to runoff; the loss to flow volume, infiltration; and the loss to peak flow, retardation. Perhaps it would be better to modify still further the definition by referring to the "effective" maximum storm. It is more important to note the differences in net effect between storms producing equal volumes of rainfall, in which case differences in distribution through time and over area result in flood hydrographs of varying shape and peak discharge. The apparent inability to sustain excessive intensities over any but relatively small areas would indicate the impossibility of experiencing concurrent maximum precipitation over the watersheds of every tributary of a basin except in the case of a very small one. The critical synchronization of tributary flow, like the most critical storm rainfall pattern, will depend largely upon the characteristics of the particular basin. On the larger basins, synchronization is involved in the direction of storm movement. This is not to be confused with the so-called "storm track" comprised of the trace of movement of an area of low barometric pressure. In certain localities physiographic controls will establish a fairly stable régime in the direction of movement of the "rain front" of "rain pattern." More often, the direction is fortuitous, depending usually on the orientation of an extended quasi-stationary front and the timing of energy releasing "waves" along it. When physiographic influences do not predominate and the region of homogeneity is sufficiently extensive to permit the transposition and superposition of storm maps to and upon basin maps,

NOTE.—This paper by Merrill Bernard, M. Am. Soc. C. E., was published in January, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1943, by Edgar Dow Gilman; April, 1943, by Messrs. Clarence S. Jarvis, Ivan E. Houk, W. G. Hoyt, and L. R. Beard; June, 1943, by Messrs. Robert E. Kennedy, and L. K. Sherman; October, 1943, by Messrs. C. H. Eiffert, and Gail A. Hathaway; and November, 1943, by L. Standish Hall.

[13] Hydrologic Director, U. S. Weather Bureau, Washington, D. C.

[13a] Received by the Secretary December 23, 1943.

all, or the greater number, of the steps given in Table 5 are necessary to the synthesis of the flood hydrograph.

Mr. Jarvis' particularly comprehensive discussion warrants far more consideration than can be given to it in these closing paragraphs. His questions

TABLE 5.—STEPS IN FLOOD HYDROGRAPH SYNTHESIS

| No. | Step | Means of determination |
|---|---|---|
| 1 | Maximum storm volume | Total isohyetal map; area-volume relations |
| | Variation of Rainfall Depth— | |
| 2 | With time | Mass curves of rainfall |
| 3 | With duration | Depth-duration relations |
| 4 | With area | Depth-area relations |
| 5 | Distribution of rainfall depth over area | Isohyetal pattern, actual or synthetic |
| 6 | Critical time increment | Equal to or multiple of basin's concentration period |
| 7 | Critical time pattern | Arrangement of increment depths in critical time order |
| 8 | Maximum runoff volume | Superposition of total storm to give greatest volume of runoff under minimum loss conditions, to include the snow-melt increment when appropriate |
| 9 | Critical concentration of runoff | Superposition of storm increments to give greatest concentration of runoff in critical period |
| 10 | Critical synchronization of tributary flow | Progressive routing of tributary flow to disclose critical storm pattern and position |
| 11 | Maximum flood peak | Most effective combination of conditions for critical concentration of runoff and critical synchronization of tributary flow |

in regard to the apparent inadequate values of $W_E$, effective precipitable water, are answered in references (2), (3), and (38).[13b]

Mr. Hoyt has referred to the progress report (30) of the special Society Committee on Meteorological Data. This piece of constructive criticism, to the writer's knowledge, has contributed much in guiding the Weather Bureau to its present important place as a source of engineering data. However, in retrospect, a principle in democratic evolution seems to have revealed itself; namely, the need must be demonstrated before fulfilment is made possible through financial support. Engineers have always wanted more precipitation data. The Weather Bureau (on appropriations of less than 5¢ per capita) was trying to provide them; but it took the demonstrated needs of flood control and soil erosion prevention to bring about the approval of Congress for the establishment of a network of nearly 3,000 recording (intensity) rainfall stations, several hundred badly needed stream discharge stations, and a score of field and indoor laboratories—all now playing important rôles in engineering programs.

Mr. Houk's comments on the distinction to be made between the maximum storm and the maximum flood emphasize those of Mr. Gilman. In general, the magnitude of storm and flood is of comparable order as the upper limits are approached. On small watersheds, rainfall must be expressed in terms of intensity for short periods. On large basins the volume of rainfall produced by an extensive storm will determine the magnitude of the resulting flood. In the former case, there will be a considerable fluctuation in ground surface effects, thereby disrupting the frequency relations between rainfall and the

[13b] Numerals in parentheses, thus: (2), refer to corresponding items in the Bibliography at the end of the paper (see Appendix 1), and at the end of discussion in this issue.

flood. In the latter case, initial rains will have reduced infiltration rates and will have provided sufficient water to occupy pondage and channel storage fully. Thus, the resulting flood will bear a far more consistent relation to storm rainfall than the rainfall of a brief storm over a small area.

The difficulties in determining storm rainfall over a prescribed basin through the solution of an equation of flow are partly revealed through an analogy in which the problem is to determine the deposition of suspended material in a silt laden stream throughout a reach in which the slope has been abruptly flattened. The flow is confined within an unyielding channel of fixed dimensions, and all the principal factor values are determinable, including the volume of deposited material and the residual of silt which leaves the reach at its outlet. In the equation of moist air flow, adequate means for measuring and giving quantitative expression to the comparable variables are largely lacking. The imaginary boundaries of the flow channel beyond the assumed inflow section have no assurance of reality. The residual moisture leaving the basin is not subject to measurement. Improved instrumentation and a more dense upper-air observation network can be expected to overcome most of these deficiencies.

It is fairly simple to determine the prevailing direction of inflow and, therefore, the basin constant $K$. Gradient winds can likewise be estimated with fair accuracy. The value of $R$ can be found by planimetering the isohyetal map. It is not always possible, however, to disassociate $v$ and $W_E$ in actual storms. This does not invalidate the basic method of computing the maximum rainfall amounts since, for the physical upper limit, it would be necessary to assume that these two factors combined at their respective maximum effectiveness. It is not altogether logical to combine them at their physical upper limits because they are not completely independent.

In order to develop and maintain maximum velocities (which increase with height and, therefore, must be integrated throughout the depth of the inflow layer to provide maximum rates of inflow), there must be steep gradients of temperature as well as of pressure. At the same time the maximum moisture charge must necessarily be contained in a broad tongue of moist air characterized by the maximum possible saturation temperature at all levels. Also, the moisture charge must be horizontally isothermal throughout the full width of the tongue. Thus, the introduction of a temperature gradient automatically places limitation on the moisture charge.

Mr. Houk's suggestion that values for $K$, $v$, and $W_E$ for the outstanding storms of record be published strikes a note of accord among all engineers interested in hydrometeorology. An illustration (38) is given in the case of the Sacramento River Basin where the principal lifting agencies providing the mechanism for producing rainfall are the Coastal Ranges and the Sierra Nevada. Since these barriers remain stationary, the problem of determining rainfall amounts through computation is somewhat simplified. The Point Reyes (Calif.) station was found to be representative for inflow wind velocities, and the dew points at San Francisco (Calif.) were used as an index to the moisture charge of the inflowing air mass. It was possible, therefore, to complete the computations of rates of rainfall by use of the basic formula,

$R = \int K v W_E \, dt$ (see Eq. 2). Fig. 27 compares the theoretically computed average depths of rainfall over the basin with the observed depths obtained by planimetering isohyetal maps. Fig. 28 shows a comparison of the computed and observed time-intensity patterns for the storm of February 25–29, 1940.

The writer can agree with Mr. Beard in his principles of relative accuracy, the validity of which must await the time when observations of the variables

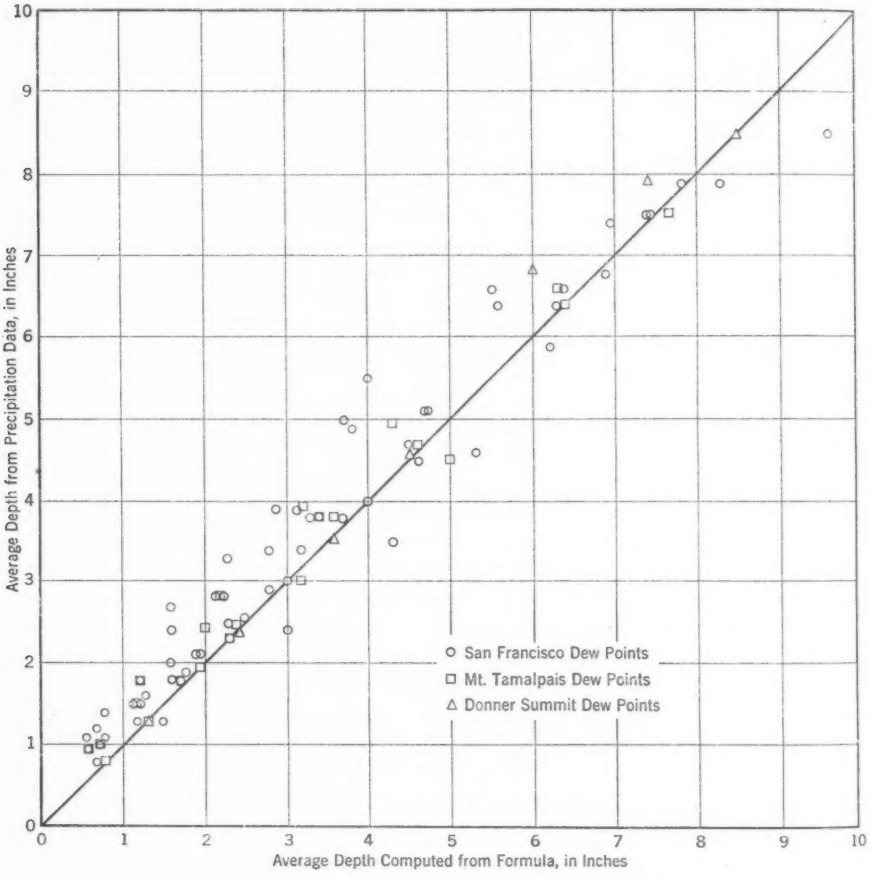


FIG. 27.—AVERAGE PRECIPITATION DEPTHS OVER TOTAL BASIN

he has enumerated have covered a period sufficiently long to have statistical significance. Engineers are now merely groping toward a state of knowledge that will ultimately introduce reasonably precise procedures into hydrologic design. In the meantime, efforts to refine the knowledge on any one of the factors in the sequence of events from the "meteorological storm" through the "hydrologic storm" to the culminating flood would seem warranted.

The designation of "maximum possible" to the storm values discussed in the writer's paper has the intended meaning that such values can be approached

but not exceeded, and therefore can be considered only as the asymptotic limit of the magnitude-frequency curve. As Mr. Sherman has stated, such values should not be the basis for design in any except the case where social and psychological considerations dictate security beyond economic justification. In any case, it would seem as much an advantage to the flood control designer to know the upper limit of magnitude of his major factor, the storm, as it is to the structural designer to know the ultimate strength of his materials.

The criticism of listed regions of meteorological homogeneity, particularly as the basis for justifying storm transposition, is readily admitted. Apparently, there is a compensative circumstance in certain mountainous areas where orographic controls so stabilize the rainfall régime that the seasonal pattern

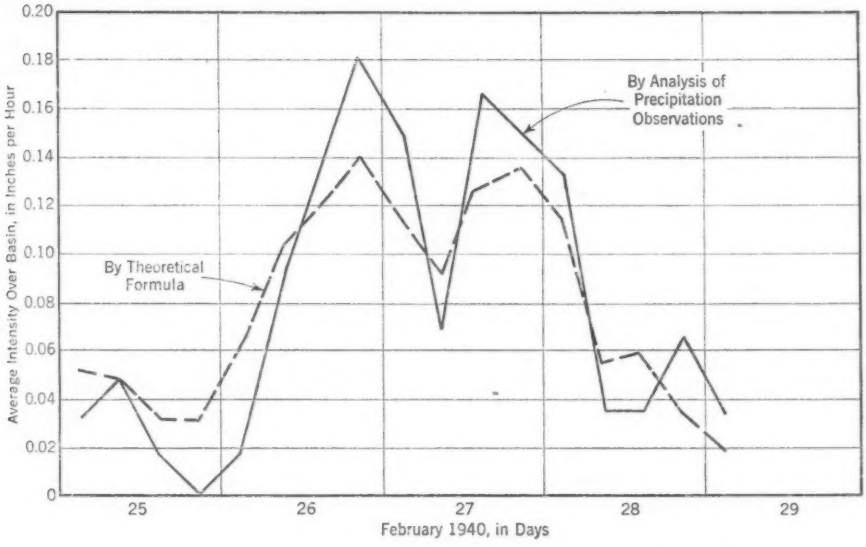


FIG. 28.—COMPUTED PRECIPITATION INTENSITY AVERAGE OVER TOTAL BASIN; STORM OF FEBRUARY 25–29, 1940

can be used as a guide to the distribution of storm rainfall over a large basin. The opinions of Messrs. Beard, Kennedy, and Hall have not overemphasized the cautions to be observed in storm transposition.

The writer is indebted to Mr. Kennedy for his review of the work of Messrs. Showalter and Solot (20). The theoretical determination of storm rainfall could be given only a limited place in the original paper.

It is hardly correct to say that storm transposition is being supplanted by area-duration-depth data. The designer will always resort to the simple expedient of superimposing storms over basins wherever homogeneity of rainfall régime exists. Values taken from area-depth curves are meaningless until they have been expressed as areal patterns—either actual or synthetic. Mr. Eiffert's reference to the apparent inconsistency in the title of Table 1 is noted. Area-depth curves constructed on a single graph for successive cumulative periods of time from the beginning of the storm are accurately referred

to as time-area-depth curves, whereas those for separate durations, with no common beginning time, are more appropriately called duration curves.

In the work of the Hydrometeorological Section of the Weather Bureau, it has been found that the staggered hours of observation have proved blessings in disguise, disclosing as they do the times of beginning and ending of rain (39).

Mr. Hathaway has made a particularly valuable contribution to the paper in illustrating the steps toward design objectives beyond those involving meteorology. That pertinent storm data will ultimately be made available in the concise terms characteristic of the Corps of Engineers should be appreciated by the civil engineering profession.

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(3) **A Report on the Maximum Possible Precipitation over the Ohio River Tributary Basins above Pittsburgh, Pa.,** Hydrometeorological Section, U. S. Weather Bureau, in cooperation with the U. S. Engr. Dept., Corps of Engrs., June, 1941.

(20) **"Computation of the Maximum Possible Storm,"** by A. K. Showalter and S. B. Solot, *Transactions*, Am. Geophysical Union, 1942, Pt. II, pp. 258–274. (*a*) Fig. 2, p. 261. (*b*) p. 259.

(30) **"Meteorological Data,"** *Proceedings*, Am. Soc. C. E., Vol. LIX (1933), pp. 153–182.

(38) **"Maximum Possible Precipitation Over the Sacramento River Basin of California,"** *Hydrometeorological Report No. 3*, Hydrometeorological Section, U. S. Weather Bureau, in cooperation with the U. S. Engr. Dept., Corps of Engrs., May, 1942, p. 113.

(39) **"Advantages in the Non-Uniform Hour of Observation in the Interpretation of Published Precipitation Data,"** by J. R. Miles, *Monthly Weather Review*, April, 1940, p. 99.

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DISCUSSIONS

# THE QUEENS MIDTOWN TUNNEL

## Discussion

BY A. A. EREMIN, AND ARTHUR S. TUTTLE

A. A. EREMIN,[22] ASSOC. M. AM. SOC. C. E.[22a]—Interesting details of design and construction for the Queens midtown subaqueous tunnel have been presented in this paper. In the "Conclusions" the author has stated that "Extended research is no longer necessary in planning and designing tunnels for highway traffic, * * *." Although it is true that considerable improvement in design and construction of subaqueous tunnels has occurred in the last two decades, further study is necessary.

*Method of Construction.*—It is well known that in tunneling soft ground shield driving is the most economic method. The protective blanket is generally used to prevent excessive loss of air and blowout accidents although the mechanical properties of the blanket assuring safety are still not definitely determined. On account of defects in the protective blanket, a serious blowout accident occurred during the construction of the Holland Tunnel. As a result, an improved type of blanket was used in the East Boston Tunnel, which, nevertheless, suffered considerable loss of air through the pile holes at the pierhead line. Similarly, much air was lost during construction of the Queens Midtown Tunnel, as reported by Mr. Singstad.

The Posey Tunnel penetrating soft ground between Oakland and Alameda, Calif., although designed almost at the same time as the Holland Tunnel, was constructed by open-trench excavation and by floating pre-cast concrete segments. The shield-driving method was rejected because of the excessive cost of protective blanket.

Shield driving has many advantages, as has been pointed out in this paper:

(1) It causes very little disturbance in the ground. This is especially important because subaqueous tunnels are generally constructed at highly de-

NOTE.—This paper by Ole Singstad, M. Am. Soc. C. E., was published in March, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1943, by Messrs. Thomas W. Fluhr and Howard L. King; June, 1943, by Messrs. Francis V. Wagner, William J. Wilgus, and S. A. Thoresen; September, 1943, by Messrs. H. G. Cruthers, Leo Geenens, Charles E. Conover, Edward Levy, and Robert F. Legget; and November, 1943, by Orrin L. Brodie.

[22] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[22a] Received by the Secretary November 29, 1943.

veloped sites, and slight disturbance in the ground permits construction of twin tunnels close together. The Queens Midtown tunnels were 45 ft apart. Similar spacing of twin tunnels proved safe in driving the Holland and Lincoln tunnels.

(2) It may often result in saving of construction time. The speed of driving for the Queens Midtown tunnels was about 7.4 ft per day. The Lincoln tunnels were driven at the even higher rate of 45 ft per day.[23] Nevertheless, because it is an essential part of shield driving, further study of the mechanical properties of the protective blanket is highly desirable.

*Loading.*—An extensive study of loading on the subaqueous tunnel lining was made for the design of the Holland and Posey tunnels. As a result of this research, a circular rigid tube with steel ties at the roadway slab and ceiling was adopted for lining tunnels. During the construction of the Posey Tunnel at some sections the tie rods at the roadway slab were under compression instead of tension. The horizontal diameter was shortened during construction of the Lincoln Tunnel.[23] In the Queens Midtown Tunnel a flattening effect in the heavy cast-iron lining was observed, thus confirming the original study of loading. Mr. Singstad has shown important effects of grouting on distribution of loading and increasing resistance of the ground to the air losses. However, he states that (see heading, "Construction: River Tunnels") "The heading was dewatered on August 8, 1938, disclosing an inflow of grout and muck at a maximum depth of 14 ft at the shield and running back a distance of 208 ft in the tunnel." Thus, in shield driving, a further study of the methods of grouting is still essential. Correct grouting may control the load on the lining and the rate of shield driving as stated in the paper.

In the shield-driven tunnels various types of lining have been used. The most well-known linings are cast-iron, structural steel plate, and concrete. Mr. Singstad states that (see heading, "Design of Structures: Lining for Shield-Driven River Tunnels") "Cast-iron lining of the usual design was chosen for the shield-driven tunnels as best adapted to resist shield thrusts and to provide rigidity and watertightness" and that "* * * lack of uniformity in stress, may contribute to some leakage, * * *." The cast-iron segments have been proved rigid. However, their rigidity is decreased at the connections. Fig. 3 shows that the bolt holes in the segments are $\frac{1}{4}$ in. larger than the bolts. Evidently with such clearance pre-stressing and the use of a high strength steel for the bolts cannot make the connections as rigid as the segment units. Sufficient rigidity and watertightness in the lining of the Lincoln and Detroit tunnels were obtained with the structural plate segments, which are generally three times lighter than cast-iron segments. To resist the direct thrust from the shield the structural plate liners generally are reinforced with old rails or H-sections.

*Ventilation.*—When the Holland Tunnel was being designed, very little was known about the ventilation of vehicular tunnels. In cooperation with the U. S. Bureau of Mines, formulas for the design of ventilation features in the tunnels were developed. Poisonous gas, carbon monoxide, was limited to 4 parts per 10,000. After the construction of the Holland Tunnel tests revealed

[23] *Engineering News-Record*, June 24, 1937, p. 955.

that during maximum traffic the amount of carbon monoxide never was more than 1.60 parts per 10,000.[24] At first it was concluded that the Holland Tunnel and the Posey Tunnel (which was designed on a similar principle) were over-equipped. However, it was decided later that greater circulation of fresh air purifies the odor and smoke resulting from combustion of oil in the vehicles. Nevertheless, in subsequent tunnels and in the Queens Midtown Tunnel, design of ventilation facilities was based on a limitation of the maximum amount of carbon monoxide. Evidently the physical properties of smoke and burned oil are different from those of the vitiated air containing carbon monoxide. Therefore, for an economic ventilation design, further study of the smoke problem is necessary. Separate fresh air outlets were used in the Broadway Tunnel in Berkeley, Calif. It is quite possible that the separate openings may require a greater air pressure at the bulkhead. For efficient design of ventilation a minimum loss of air pressure at the ventilating fans and at the outlets near the bulkhead is essential.

*Transition of Light.*—Various methods of lighting at the portals have been used to eliminate the discomfort and the blinding effect of daylight. It is interesting to determine the various possible structural details for regulating light. Mr. Singstad has shown that the belled-out approach structure provided a gradual transition of light. In the Broadway Tunnel in Berkeley, transition of light was improved with transparent roofing between the approach retaining walls.

*Conclusions.*—Vehicular tunnels have an important rôle in solving problems of growing interurban traffic. Therefore, the author should be highly commended for presenting such interesting information on the design and construction of the Queens Midtown Tunnel.

ARTHUR S. TUTTLE,[25] PAST-PRESIDENT AND HON. M. AM. SOC. C. E.[25a]—Essential data concerning the Queens tunnel project, with very complete information as to its inception, administration, design, construction, and cost are recorded in this paper. The profession is indebted to Mr. Singstad for making this information available not only as a matter of historical interest but also as a help in the planning of structures of similar character.

As is generally the case with all monumental projects undertaken by the City of New York, many years elapse between the need for such a facility and the completion of the final plan, during which period various pros and cons are given consideration, all of which must be appraised. The possibility of securing federal aid in 1936 was doubtless responsible for the city committing itself definitely to the project at that time. The 30% grant, with a 70% loan offered by the PWA, for meeting the entire cost of the project, was conditioned upon the presentation of evidence that the project was necessary and would be self-supporting and that construction would be begun without delay. The former condition was intended to insure against wasteful expenditures; and the latter,

[24] *Engineering News-Record*, June 9, 1927, p. 934.

[25] Cons. Engr. (Tuttle, Seelye, Place & Raymond), New York, N. Y.

[25a] Received by the Secretary December 23, 1943.

to relieve unemployment in accordance with the primary purposes of the congressional act under which the PWA was set up and provided with funds.

The tunnel is located between the nonrevenue producing Williamburg and Queensborough bridges, both of which are badly congested during hours of maximum traffic. Extensive traffic studies were presented by the Tunnel Authority to support the claim that the relief to the bridge traffic which would be afforded by the tunnel would bring enough business to justify the project as financially sound. These studies contemplated the provision, by the city, of adequate highway approaches in the Borough of Manhattan and the Borough of Queens, which were considered essential to the successful operation of the tunnel. Had the cost of these approaches been included in the project, it is more than likely that the project would have been disqualified for a loan on the ground that it would not be self-supporting.

Although plans have been developed for a much needed crosstown tunnel in Manhattan to relieve the narrow and congested streets, overworked by traffic destined for the west side and for New Jersey, no definite commitment has yet been made. In the Borough of Queens a part of the approach system has been constructed, but major links are still lacking. Until adequate feeders are provided, the traffic capacity of the tunnel will not be fully utilized, and revenues will be handicapped.

When the project was finally accepted by the PWA as a part of its program and the "go-ahead" signal was given to the Tunnel Authority, difficulties began to develop: State legislation was threatened which would substitute a bridge instead of a tunnel; civil service procedure for a time raised barriers to the selection of an engineering staff properly qualified for the work; and rapidly rising prices for labor and material occasioned a question as to the sufficiency of the available funds. Pending the solution of these difficulties, the federal government threatened to withdraw the loan and grant so that the funds could be diverted to projects which could be put under construction more speedily. During all of this troublous period, there was very close cooperation between the engineering staff of the Tunnel Authority and the engineering staff of PWA, the final outcome being a commitment to the river crossing, which necessarily placed the project on a list from which there could be no withdrawal.

The tunnel, as completed, undoubtedly represents the latest and best of engineering skill in subaqueous tunnel construction, and the author is to be complimented for his success in overcoming the many obstacles placed in his way as the project advanced.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

# SIMPLIFIED ANALYSIS OF SKEWED REINFORCED CONCRETE FRAMES AND ARCHES

## Discussion

BY J. CHARLES RATHBUN

J. CHARLES RATHBUN,[21] M. AM. SOC. C. E.[21a]—There is a tendency among engineers to avoid reinforced concrete skew arches of the barrel type even when this type of structure would be more economical and would give a better alinement than the type adopted. Some designers have even used skew arches of rib construction with the floor slab cast integral with the rib. One such bridge was built with an added slab at the intrados. Of course those who try to dodge the problem not only fail in their purpose but introduce other problems, of a more difficult nature, in analysis.

Since 1924 a number of papers on the analysis of the skew arch have been published by the Society.[3, 6, 22, 23] These with their discussions give a complete view of the problem and its solution. Any engineer entrusted with a problem for which a skew arch would make a satisfactory solution owes it to his client to review this literature. A solution is offered for arches with fixed abutments.[3] The equations for hinged abutments were first developed by the writer in a report to the Westchester County Park Commissioners in 1925. Later they were published, along with the method of computing the steel reinforcement.[23]

In offering a practical application of that method, Mr. Hodges' paper is a distinct contribution to this series of papers. He has used it in the design of many skew arches for the past twenty years. It is hoped that this paper will impress the profession with the fact that an analysis of a skew arch can be made and that the stresses in the finished structure can be anticipated with design accuracy and at a reasonable cost.

NOTE.—This paper by Richard M. Hodges, M. Am. Soc. C. E., was published in May, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by Messrs. C. D. Geisler, and Arthur G. Hayden; and November, 1943, by Bernard L. Weiner, and Alfred L. Parmé.

[21] Prof., Civ. Eng., College of the City of New York, New York, N. Y.

[21a] Received by the Secretary January 7, 1944.

[3] "Analysis of the Stresses in the Ring of a Concrete Skew Arch," by J. Charles Rathbun, *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

[6] "Design of a Reinforced Concrete Skew Arch," by Bernard L. Weiner, *ibid.*, Vol. 96 (1932), p. 1212.

[22] "Crown Stresses in a Skew Arch," by J. Charles Rathbun, *ibid.*, Vol. 94 (1930), p. 135.

[23] "An Analysis of Multiple-Skew Arches on Elastic Piers," by J. Charles Rathbun, *ibid.*, Vol. 98 (1933), p. 41.

The writer proved[24] that the "Rankine theory" (in which the force lines are parallel to the spandrel walls and stress distribution is uniform along the abutments) is very nearly correct for symmetrical arches loaded with symmetrically placed vertical loads. This is the condition for dead load and full uniform live load for symmetrical skew arches. The Rankine theory is not correct (often it is far from correct) for unsymmetrical arches, for unsymmetrical vertical loads, and for horizontal loads such as earth pressure and forces due to shrinkage and temperature.

Soon after the publication of the writer's solution to the problem, A. G. Hayden, M. Am. Soc. C. E., suggested that the work of analyzing skew arches might be shortened by developing equations for the deviation of the stresses from those obtained by the Rankine theory. With these equations the preliminary analysis could be corrected. Mr. Hodges has used somewhat the same idea with the right arch as the basis of his work. The writer would have preferred to see Mr. Hayden's method used rather than Mr. Hodges' method because the magnitude of the corrections would have been less. Theoretically, the choice is of little importance, however, as both methods are correct. Perhaps Mr. Hayden's suggestion led to more involved equations and Mr. Hodges has found his solution more practicable.

The solution presented by the writer in 1924 has had considerable criticism because it does not conform to a preconceived idea that the "stresses take the shortest route to the abutments." The writer has yet to see a sound quantitative analysis of this tendency. Many tests have been made to show that the idea is incorrect. Three such tests have been made by the writer.[22,25]

To test the correctness of the theory the Society had the late George E. Beggs, M. Am. Soc. C. E., conduct a series of tests on four arches. These arches all had fixed abutments and were tested for vertical loads. The reactions for these same loads were computed by the elastic theory under the writer's supervision. Contours of the influence surfaces of the components of the abutment reactions were drawn up for each arch and the two methods compared.[10] Models of these surfaces (see Fig. 8) were also made to assist in this comparison. The tests and the computations were in virtual agreement and the committee reported,[26]

> "For the practical design of skew arches, the limited number of tests which have been conducted indicate that Professor Rathbun's theory may safely be applied for the computation of the six components of the abutment reactions."

After these are computed the problem is a simple one in statically determinate stress analysis.

The models used by Professor Beggs were of rubber with the very high Poisson's ratio of 0.34, compared with about half this value for concrete. The committee decided that the discrepancy "would change the results of the cal-

[24] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 620.

[25] *Ibid.*, Vol. 98 (1933), pp. 26 and 66.

[10] "Concrete and Reinforced Concrete Arches," Final Rept. of the Special Committee, *ibid.*, Vol. 100 (1935), pp. 1551–1573.

[26] *Ibid.*, p. 1572.

culations by only a small amount." [26] In fact the model surfaces showed that the change would occur in but few areas and for loads over most of the arch the effect would be very little. Designers of right arches, where the same effect occurs, have neglected to take Poisson's ratio into consideration, with no serious consequences.

The writer believes that the report of this committee has convinced the most skeptical engineers that the elastic theory, as developed, is sufficiently correct for design purposes.

One reason that the elastic theory, as developed for the skew arch, has not been popular is probably the writer's fault. In the analysis he used a geo-

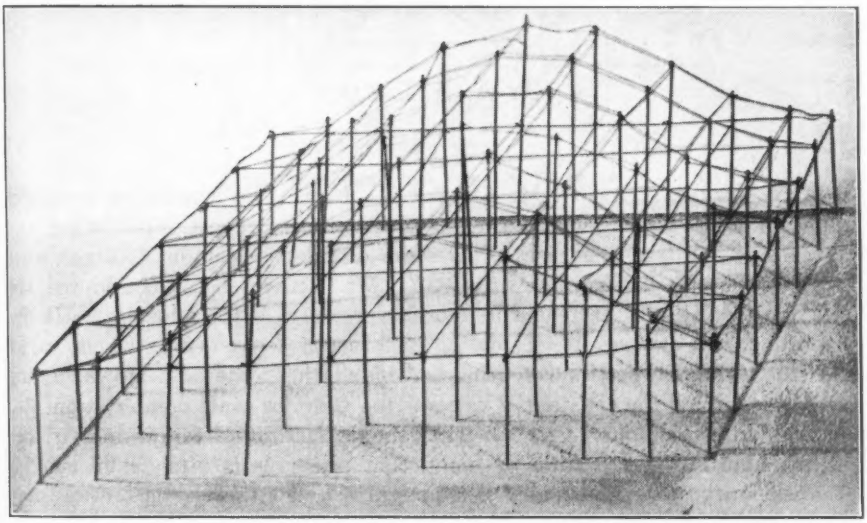

FIG. 8.—MODEL OF INFLUENCE SURFACE

metrical approach instead of the method of virtual work; he includes horizontal and radial shear deformations as well as shortening due to thrust; he also developed the deformation due to the several components of the moments. Many of these terms have since proved to be negligible. He covered both the symmetrical and unsymmetrical skew arch for all types of loading. The result was that the paper became long, cumbersome, and laborious to follow. One critic stated that "No arch can follow that much mathematics," to which Professor Beggs replied, "Four Princeton arches did." Mr. Hodges seems to have overcome this objection.

With this paper of Mr. Hodges the question of the design of a reinforced concrete barrel skew arch should present little difficulty to those familiar with the analysis and design of the right arch of the same type. It is hoped that the paper will renew interest in the subject and encourage engineers who have arches to design to familiarize themselves with the basic theory and the literature on this subject.

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DISCUSSIONS

# A SYMMETRICALLY LOADED BASE SLAB ON AN ELASTIC FOUNDATION

## Discussion

BY A. A. EREMIN

A. A. EREMIN,[14] ASSOC. M. AM. SOC. C. E.[14a]—The foundation pressures under an elastic footing vary according to the elastic properties of the soil and the footing. Interesting cases of pressure distributions under footings were illustrated by D. P. Krynine[15] in 1937. Mr. Benscoter has considered the case of a footing with decreased foundation pressures at the midspan. He has determined the stresses in the footing in terms of a constant $\alpha$ which varies with the elastic properties of footing and foundation material. However, the range over which the constant $\alpha$ varies is too wide for exact computation.

The writer has found that the approximate method of computing stresses for the assumed distribution of foundation pressures is often sufficient for practical purposes. The foundation pressures under the elastic footing may be assumed to vary according to the sine curve in Fig. 14. The average foundation pressure in Fig. 14 is $w_o$ and the reduction of foundation pressure at the midspan is $w_1$. Assume that the footing slab in Fig. 14 is supported by hinged supports at the ends; then the bending moment at midspan will be

$$M = \frac{L^2}{8}(w_o - 0.41\, w_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (48a)$$

If the footing slab in Fig. 14 is supported by built-in ends, the bending moments at the built-in ends are

$$M' = \frac{L^2}{12}(w_o - 0.30\, w_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (48b)$$

NOTE.—This paper by Stanley U. Benscoter, Jun. Am. Soc. C. E., was published in May, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by Messrs. Robert O. Jameson, and L. J. Mensch; and November, 1943, by Alfred L. Parmé.

[14] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[14a] Received by the Secretary December 10, 1943.

[15] "Pressures Beneath a Spread Foundation," by D. P. Krynine, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 838.

The example shown in Fig. 8 may be solved by applying Eqs. 48 as shown in Figs. 14 and 15. The average foundation pressure in the footing slab (Fig. 14) is $w_o = 3$ kips per ft. Assume that the foundation pressures vary according to the sine curve, with a reduction of pressure at the midspan to $w_1 = 2.1$ kips

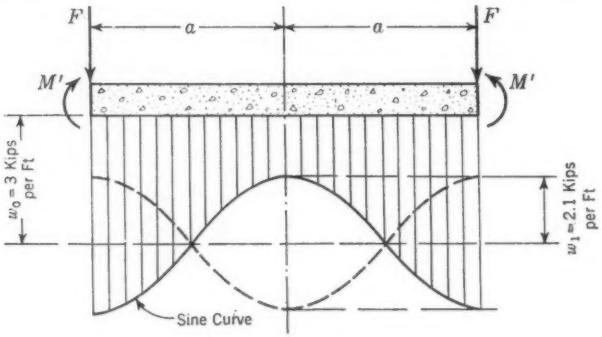


FIG. 14.—FOUNDATION PRESSURE DIAGRAMS

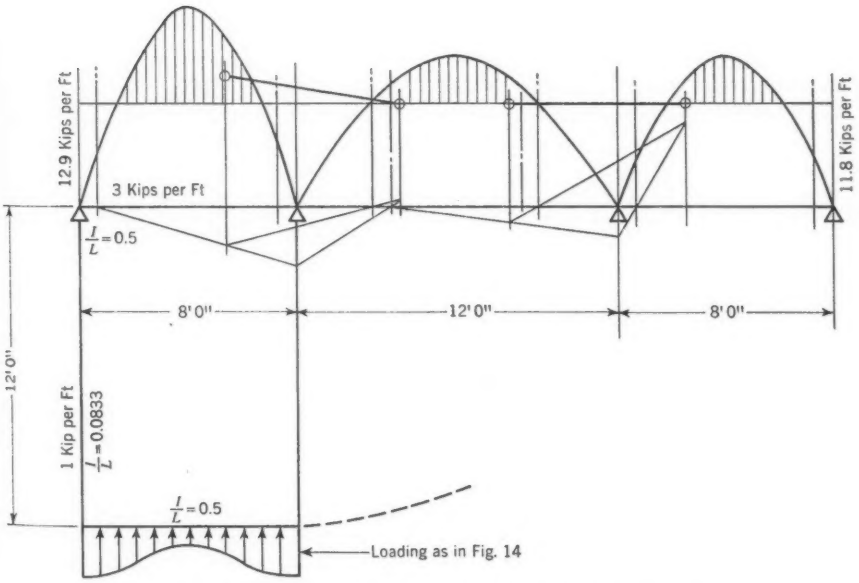


FIG. 15.—DISTRIBUTION OF MOMENTS IN RECTANGULAR CONDUIT

per ft, or about 70% of the average pressure. The assumption that foundation pressures are reduced at the midspan may be verified easily by tests. The bending moments at the fixed ends of the footing slab (Fig. 14) computed by Eq. 48*b* are $M' = 12.65$ kip-ft. The bending moments in all members of the rigid frame were determined by the graphical method[16] as shown in Fig. 15.

[16] "Analysis of Continuous Rigid Frames by Graphical Distribution of Moments," by A. A. Eremin, published privately by the author, 1943.

The difference between the resulting bending moments and those shown by the author is negligible.

Often the foundation pressures at midspan of the footing slab are increased, as shown by Professor Krynine.[15] In this case also the foundation pressures may be assumed to vary according to the sine curve, with the maximum pressure at the midspan, as shown by the dotted line in Fig. 15. Bending moments in the footing, in this case, may also be determined by Eqs. 48, except that the sign before the second term in the parentheses will be changed from minus to plus.

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DISCUSSIONS

# RAINBOW ARCH BRIDGE OVER NIAGARA GORGE

## A SYMPOSIUM

### Discussion

BY C. M. SPOFFORD, L. J. MENSCH, AND O. H. AMMANN

C. M. SPOFFORD,[28] M. AM. SOC. C. E.[28a]—The Rainbow Arch Bridge is a beautiful structure, perfectly adapted to its location. Great credit is due to the engineers and architects who evolved the design, to the men who built the bridge so successfully and expeditiously, and to the writers of the papers in this Symposium for their comprehensive description of the design and construction.

Although the laboratory work on the two-hinged segmental arch is valuable in itself, its publication as a part of the Symposium is doubtless due to the fact that it was of assistance to the designers in determining whether or not to use two-hinged arches or perhaps to confirm their opinion that a two-hinged arch would not be satisfactory for this bridge.

The use of the expression "classic arch theory" seems ill-advised, especially as applied to a two-hinged parabolic arch. The writer knows of no classic arch theory for two-hinged arches. Such an arch has only one indeterminate statical force; namely, the horizontal component of either reaction. An expression for this can be readily developed by the conventional method of assuming one of the hinges to be on rollers and equating the expression for its horizontal deflection under any load, with the deflection of the same point due to a horizontal unknown force applied at the hinge. Certain assumptions may be made in a preliminary computation, a simple equation developed, the deflection of the arch computed, and the computation revised, if it seems advisable, by dividing the arch into sections and using summations instead of integrations. The experimental results would have been more useful to arch

NOTE.—This Symposium was published in October, 1943, *Proceedings*. Discussion on this paper has appeared as follows: December, 1943, by Egidio O. Di Genova, and Charles Mackintosh; and January, 1944, by C. M. Goodrich, and T. Kennard Thomson.

[28] Hayward Prof. of Civ. Eng., Emeritus, Mass. Inst. Tech., Cambridge, Mass.; Cons. Engr. (Fay, Spofford & Thorndike), Boston, Mass.

[28a] Received by the Secretary January 3, 1944.

designers had they been expressed in terms of the horizontal component of a reaction.

In designing two-hinged parabolic arches, both for bridges and buildings, the writer has found no serious technical difficulties, especially when advantage is taken of the properties of the position line and the kernel points.

More experimental work might well have been done to determine the three unknowns of a hingeless arch instead of making the "limited observations upon this type of arch" of which no record appears in the Symposium.

Congratulations should be extended to the engineers of the Bethlehem Steel Company for their unusually thorough investigations and for their great care in the erection procedure. The thanks of the profession are due especially to the authors of the papers on fabrication and erection for their thorough treatment of these phases of the construction. It is worthy of note that safety nets were used and that no fatalities occurred during the construction of the bridge—an unusual record. Another interesting feature is that the erectors used capsules in order to establish the total crown forces and concluded that hydraulic jacks are reasonably accurate when applied forces are measured on the increasing side of the reaction. It is hoped that in the closures a detailed description of these capsules will be given. The writer's experience with hydraulic jacks for determining reactions has not been entirely satisfactory. He notes with interest that the jacks were calibrated in a testing machine which had been previously calibrated by the use of proving rings and assumes that it was not found convenient to use proving rings for this bridge. The writer has used proving rings on various statically indeterminate structures.[29]

The flexibility of the arch ribs is not surprising; the same thing was noticed by the writer during the construction of a plate girder roof arch—a horizontal adjustment at one end of which was made without difficulty by several laborers pushing the arch end horizontally.

An unusual feature of this bridge is the use of steel keystone pieces and the method of introducing them into the arch ribs. This is the first time, so far as the writer knows, that such a device has been used, although one important reinforced concrete arch bridge, designed by the writer's firm, was erected as a three-hinged arch and made two-hinged by closing the center hinges after erection.

One of the factors that is always of great interest to engineers is that of cost—either the total cost divided between foundations and superstructure, or, if let at a unit price basis, the pound price of both silicon and carbon steel. It is hoped that the authors, in their closures, will give such information.

The writer assumes that consideration was given by the designers to the advisability of using a light-weight floor and that such a floor—either light-weight concrete with bituminous surfacing or steel grating—was found uneconomical. It would be interesting to have this matter presented in the authors' closures.

[29] "Weighing Bridge Reactions with Proving Rings," by C. M. Spofford and C. H. Gibbons, *Engineering News-Record*, March 28, 1935, p. 446.

L. J. MENSCH,[30] M. AM. SOC. C. E.[30a]—The Rainbow Bridge is one of the world's outstanding bridges, and the practical bridge engineer will greatly appreciate receiving information as to the high points of its design, fabrication, and erection. The many illustrations, the history of the design, Table 1, and the summary of quantities are very interesting and one nearly lost feature of the art has been disclosed; namely, the advisability, from the fabricating standpoint, of designing the center line of the arch rib as a series of multicentered circular arcs. The first masonry and the first metal arches were built of single-centered arcs. About 200 years ago French engineers introduced three-centered arcs, and about 50 years ago five-centered arcs made their appearance. In this bridge there is an eleven-centered arch. It is questionable whether so many centers were really necessary. Although a bridge engineer will note with respect the mathematics and the practical features of the erection problems, he probably would have liked more information on the evidently highly successful job of surveying the bridge site before and during erection and also more details on the concrete deck, the forms used to avoid an accident similar to that which occurred on the Golden Gate Bridge,[31] the kind and spacing of the expansion joints, the kind of railings used, the system of lighting, and other smaller but important features of a successfully constructed bridge. No statement has been made regarding the stiffness of the bridge under traffic—a question of greatest interest for an arch rib with a depth of one eightieth of the span.

According to the Symposium authors, the unusually shallow ribs necessitated the abandonment of the "elastic theory" and the substitution of the "deflection theory." The writer supposes that the "elastic theory" is the mathematical theory of elasticity, except that certain "elastic theory" requirements are not considered:

(1) The effect of unit stresses in excess of the true elastic limit;
(2) Poisson's effect;
(3) Shear deformation;
(4) Vibrations due to loading conditions;
(5) Changes of temperature during critical loading;
(6) Variation of the modulus of elasticity in the various parts of the structure; and
(7) Deformation of the center line of the arch rib for the equation of moments and in the elastic equations.

These phenomena are clearly and completely appraised in the oldest treatise on metallic arched bridges, analyzed by the strict mathematical theory of elasticity.[32] Early writings also emphasize the computation of deflections of the arch due to rib shortening, compression, change of temperature, shrinkage (in wood arches), and faulty joints. J. A. C. Bresse warned his students not to use ribs so shallow as to cause excessive deflection due to a moving load.

[30] Civ. Engr. and Contr., Chicago, Ill.

[30a] Received by the Secretary January 4, 1944.

[31] *Engineering News-Record*, February 25, 1937, p. 313; March 11, 1937, p. 377; and April 22, 1937, p. 590.

[32] "Recherches analytiques sur la flexion et la résistance des pièces courbes," par J. A. C. Bresse, Paris, 1854.

No real, valid reason has been given in this Symposium why this sound warning has been disregarded. For comparatively large deflections, the problems are treated in textbooks on elasticity under the heading, "Elastica," an expression first used by Leonhard Euler.

The writer holds some reservations as to the validity of Eqs. 18 to 21. For example, Eqs. 18 and 19 contain only two indeterminate quantities—$H$ and $M_{oa}$. The symbol $M'_a$ refers to a statically determinate moment of the exterior forces in a simple beam and is not concerned with the missing indeterminate value of the reaction at the left springing. To ignore the latter will intensify the conditions in Fig. 12.

To design a bridge, its span, rise, width, dead load, and live load must be known. Often there is a choice of span and rise—greater spans may reduce the cost of the foundation, and greater rise may save in the weight of metal in the arch ribs. Suppose some preliminary designs have prescribed span and rise and number of ribs. The dead load of the roadway does not vary much from 150 lb per sq ft regardless of span. The weight of the ribs may be approximated from handbooks on bridge design. A nearly correct formula for the horizontal component of the thrust is given by:

$$H = \left(w_1 + \frac{\Delta w_1}{6}\right) \frac{L^2}{8 f (1 + k)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

in which $w_1$ = the dead load per linear foot on the rib, at the crown; $\Delta w_1$ = increase in dead load per linear foot at the springing, due to the spandrel columns and the greater weight of rib; $f$ = the rise of the arch; and $k$ = a numerical factor between 0.01 and 0.03 that enters the formula to account for the shortening of the arch under load.

If the permissible stress in direct compression is assumed as one half the maximum permissible stress, then Eq. 39 may be used for a close approximation of the area of metal required in the rib at the crown. The depth of the rib is governed by the live load and deflection caused by it. Modifying and simplifying the prior work of M. Bresse,[32] the writer has derived the following fundamental formulas[33] for the three statically unknown values generated by a concentrated load $P$ at any point along the elastic arch:

Vertical component of thrust, $V$, at the left abutment—

$$V = \frac{P}{L^3} (L - L_a)^2 (L + 2 L_a) \quad \ldots\ldots\ldots\ldots\ldots\ldots (40a)$$

horizontal component of the thrust, $H$—

$$H = \frac{15 P L^2_a (L - L_a)^2}{4 L^3 f (1 + k)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (40b)$$

and moment at the left springing, in a clockwise direction[33]—

$$M_{oa} = \frac{P L_a (L - L_a)^2 \left(L - \frac{2.5}{1 + k} L_a\right)}{L^3} \quad \ldots\ldots\ldots\ldots (40c)$$

[33] "The Reinforced Concrete Pocket Book," by L. J. Mensch, Neal Publishing Co., San Francisco, 1909, p. 154 and Fig. 68, p. 155.

In Eqs. 40, in addition to the notation of the Symposium, $L_a$ = the distance from the left end of the arch axis to the load $P$.

These formulas were derived originally for a very flat circular arch of constant section, and similar ones can be obtained for a parabolic arch of constant ratio $\frac{ds}{I}$. They may be used safely for arches of the Rainbow type.

For a circular flat arch, or for a parabolic arch of constant $\frac{ds}{I}$,

$$k = \frac{45\,I}{4\,A\,f^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41a)$$

For the Rainbow arch, $\frac{ds}{I}$ is not constant and Eq. 41$a$ becomes,

$$k = \frac{12.4\,I}{A\,f^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41b)$$

The bending at any point $x$, $y$ of the arch can be computed by Eqs. 40, thus: When the distance $x$ from the left end to point $x$ is less than $L_a$;

$$M = V\,x - H\,y - M_{oa} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (42a)$$

and, when $x$ is greater than $L_a$,

$$M = V\,x - H\,y - M_{oa} - P\,(x - L_a) \ldots\ldots\ldots\ldots\ldots (42b)$$

By solving these equations for every tenth point of the arch, for example, influence lines[34] can be constructed for determining the critical loading for eleven points on the arch. In the case of a uniform load the moment at the left springing can be found either by integrating Eq. 40$c$ between the limits $L_a = 0$ and $L_a = 0.4\,L$, or by computing the area of the influence line between those limits. For this case,

$$M_{oa}\ (\text{maximum}) = 0.174\,w\,L^2 \ldots\ldots\ldots\ldots\ldots\ldots (43a)$$

Of course, Eq. 43$a$ is strictly correct only for a constant value of $\frac{ds}{I}$. If $\frac{ds}{I}$ decreases to $\frac{1}{5}$ at the springing, the maximum value becomes

$$M_{oa} = 0.257\,w\,L^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43b)$$

By interpolation for the Rainbow arch where $\frac{ds}{I}$ decreases to $\frac{1}{1.5}$,

$$M_{oa} = 0.0185\,w\,L^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43c)$$

Adding 10% for impact stresses, $M_{oa} = 0.0204 \times 1{,}380 \times 950^2 = 25{,}400$ ft-kips for the Rainbow arch, compared with 28,850 ft-kips reported in Table 1, line 5. This difference may have been caused by assuming a concentrated load of 26,700 lb acting about 150 ft from the springing. The moment then is $0.07 \times 26{,}700 \times 950 = 1{,}775$ ft-kips, which, increased by 10%, makes the

[34] "The Reinforced Concrete Pocket Book," by L. J. Mensch, Neal Publishing Co., San Francisco, 1909, Figs. 69 to 75, pp. 155 and 157.

total moment 27,353 ft-kips. The latter value checks very closely the 28,850 ft-kips given in the Symposium. It is generally believed that the greatest moment and the greatest deflection from a distributed load occur at the quarter point of the span. A concentrated load at the three-tenth point produces a slightly greater moment in the arch, and a distributed load over the half span produces a larger moment at the three-tenth point than at the quarter point. Furthermore, the deflection is greater at the three-tenth point than at the quarter point.

For many years it was standard practice (and it is still a recommendable procedure) to design an arch bridge of the Rainbow type for a movable load on the half span from the springing to the crown, except that the crown section was designed for a uniform load extending over a distance $L/6$ on both sides of the crown. For a uniform load on the half span, the three statically indeterminate quantities can be obtained by integrating Eqs. 40 between the limits $L_a = 0$ and $L_a = \frac{L}{2}$. Thus,

$$V = \frac{13}{32} w L \quad \ldots\ldots (44a)$$

$$H = \frac{w L^2}{16 f (1 + k)} \quad \ldots\ldots (44b)$$

and

$$M_{oa} = w L^2 \frac{1 + \frac{11 k}{3}}{64 (1 + k)} \text{ (tension in the top of the arch)} \quad \ldots\ldots (44c)$$

The moment at the right springing is

$$M_s = - \frac{w L^2 \left( 1 - \frac{5}{3 k} \right)}{64 (1 + k)} \text{ (with tension at the bottom)} \quad \ldots\ldots (45a)$$

and the moment at the crown is

$$M_c = \frac{w L^2 k}{48 (1 + k)} \quad \ldots\ldots (45b)$$

The equation for the deflection of the elastic line may be obtained from the fundamental equation of flexure,

$$\frac{d^2 \delta_v}{dx^2} = - \frac{M}{E I} \quad \ldots\ldots (46)$$

in which, for a uniform load on the half span,

$$M = V x - H y - M_{oa} - \frac{w x^2}{2} \quad \ldots\ldots (47)$$

and, by double integration,

$$E I \delta_v = \frac{w}{192} \left( - 13 L x^3 + \frac{8 L x^3 - 4 x^4}{1 + k} + \frac{1.5 x^2 + 5.5 k x^2}{1 + k} + 8 x^4 \right) \quad \ldots\ldots (48)$$

For the point $x = 0.75\ L$,

$$\delta_v = \frac{w\ L^4\ (1 - 3.5\ k)}{6{,}144\ E\ I\ (1 + k)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (49)$$

and the movement is upward. It is important to know whether there are any moments acting in the arch rib when the bridge is uniformly loaded over the entire span. Integrating Eqs. 40 between the limits $L_a = L$ and $L_a = 0$:

$$V = \frac{w\ L}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50a)$$

$$H = \frac{w\ L^2}{8 f\ (1 + k)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50b)$$

$$M_{oa} = \frac{w\ L^2\ k}{12\ (1 + k)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50c)$$

and the moment at the crown is

$$M = \frac{w\ L^2\ k}{24\ (1 + k)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50d)$$

Also, at the crown:

$$\delta_v = \frac{w\ L^4\ k}{384\ E\ I\ (1 + k)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

In 1919 discussing a paper on this subject by the late George F. Swain, Past-President and Hon. M. Am. Soc. C. E., the writer presented[35] a new derivation of the Bresse equations, for the deflection and rotation of any element of a curved member, as follows:

$$\theta = \int_0^x \frac{M\ ds}{E\ I} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (52a)$$

$$\delta_h = -\int_0^x \frac{T\ dx}{E\ A} - y \int^x \frac{M\ ds}{E\ I} + \int_0^x \frac{M\ y\ ds}{E\ I} \ldots\ldots\ldots\ldots (52b)$$

and

$$\delta_v = \int_0^x \frac{T\ dy}{E\ I} - x \int_0^x \frac{M\ ds}{E\ I} + \int_0^x \frac{M\ x\ ds}{E\ I} \ldots\ldots\ldots\ldots (52c)$$

in which *ds* is an increment of length measured along the center line of the arch rib.

Eqs. 52 demonstrate that the deformations are linear functions of the exterior forces and reactions, showing that the law of superposition is valid for the deflections as well as for the stresses. M. Bresse uses them to determine the statically unknown reactions. Subsequent writers have introduced other methods such as the theory of virtual velocities, theory of least work, ellipse of elasticity, Maxwell's equations, Müller-Breslau's equations, slope deflection, etc., all of which lead to equations that must be simplified considerably before they reduce to the simple forms possible by the Bresse equations directly.[32] If the proper value of *M* is introduced, the writer's derivation of

[35] *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), Eqs. 17, 18, and 19, p. 660.

the Bresse equations[32] is valid for a simple beam, a continuous beam, a rigid frame, a three-hinged arch, and an arch with two hinges or no hinges.

Contrary to the implied limitations expressed by the Symposium authors, Eqs. 39 to 51 are quite sufficient for the design of an arch of the Rainbow type, and the results agree well with those given in Table 1. To particularize:

(1) From Eq. 39 and Fig. 12 the thrust at the crown is found to be $H = \dfrac{\left(7{,}250 + \dfrac{3{,}350}{6}\right) \times 950^2}{8 \times 150\,(1 + 0.011)} = 5{,}810$ kips for the dead load, as compared with 5,911 kips in Table 1.

(2) The bending moment from the dead load, at the crown, by Eqs. 50, is $M = \dfrac{\left(7{,}250 + \dfrac{3{,}360}{6}\right) \times 950^2 \times 0.011}{24\,(1 + k)} = 320$ ft-kips, as compared[35a] with 434 in Table 1.

(3) The maximum bending moment from live load, at the crown, by Eqs. 44 to 50, is $M = 0.0057 \times 950^2 \times 1{,}380 = 7{,}100$ ft-kips. From Eq. 41*a*, $M = 26{,}700 \times 950 \times 0.047 = 1{,}200$ ft-kips. Adding to the sum (7,100 + 1,200) 10% for impact gives 9,130 ft-kips, as compared with 8,700 in Table 1.

(4) The mechanics for analyzing the effect of a temperature drop in a fixed arch was presented by the writer[36] in considerable detail, in 1929. The analysis offered at that time yields, for the parabolic arch,

$$H_t = \frac{45\,E\,I\,c\,t}{4\,f^2\,(1 + k)} \qquad (53)$$

in which $c$ = coefficient of temperature expansion. The moments at the springing and crown, respectively, are:

$$M_s = H_t \frac{2\,f}{3} \qquad (54a)$$

and

$$M_c = H_t \frac{f}{3} \qquad (54b)$$

For the Rainbow arch the coefficients of $H_t$ in Eqs. 54*a* and 54*b* are 0.7 and 0.3, respectively. The deflection of the crown due to a drop of temperature is obtained easily by means of Eq. 52*c*, and very closely by the relation

$$\delta_v = c\,t\,f + c\,t\,\frac{L^2}{4f} \qquad (55)$$

For the Rainbow arch, when $c\,t = 0.0004$, $\delta_v = 8$ in. Assuming a temperature drop of 60° F and substituting other necessary values,

$$H_t = \frac{13.25 \times E \times 12{,}000 \times 144 \times 0.0004}{150^2 \times 144\,(1 + 0.011)} = 84 \text{ kips},$$

[35a] Correction for *Transactions:* In Table 1, line 4, Col. 1, change "− 434" to "+ 434."

[36] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1252.

as compared with 93 kips in Table 1; and $M_t = 84 \times 0.3 \times 150 = 3{,}780$ ft-kips, as compared with 4,255 in Table 1.

(5) The effect of an error of 1 in. should be $\frac{1}{4.55}$ of the effect of a drop in temperature of 60° F, as the latter would cause a potential shortening of $0.0004 \times 950 \times 12 = 4.55$ in.

(6) The maximum moment due to live load occurs at the three-tenth point, between points 5 and 6, Fig. 6. By Eqs. 47 to 50, $M = 0.0096 \times 1{,}380 \times 950^2 = 11{,}950$ ft-kips; and, by Eq. 41*a*, $M = 0.0596 \times 26{,}700 \times 950 = 1{,}455$ ft-kips. Adding 10% for impact to the sum, $M = 14{,}745$ ft-kips as compared with 13,470 ft-kips for point 5 in Table 1.

(7) The moment due to live load at the left springing agrees very well with Table 1, as demonstrated previously. It remains only to compare the temperature effects. From Eqs. 54, $M_t = 84 \times 150 \times 0.7 = 8{,}820$ ft-kips, as compared with 8,918 in Table 1.

(8) In view of the very flexible arch rib, the deflections due to dead load, live load, and drop of temperature are of great interest. Deducting the weight of the rib (because of the method of erection), the dead-load deflection at the crown is computed by Eqs. 50 as $\delta_v = 4.2$ in.; and, assuming a length of 270 ft loaded across the crown, the live-load deflection at the crown,[35] $\delta_v = 3.8$ in. For the temperature drop $\delta_v$ was found equal to 8 in. and the shortening of the arch rib due to live load was 0.55 in. The total $(4.2 + 3.8 + 8 + 0.55)$ is 16.55 in. By Eqs. 44, 45, and 48, the live-load deflection at the three-tenth point, $\delta_v = \frac{1{,}380 \times 950^4 \times 12^3 (1 + 6.8\,k)}{5{,}920\,E\,I\,(1 + k)} = 7$ in. when the half span is loaded and a constant $\frac{ds}{I}$ is assumed. Because of the larger moment of inertia near the springing the actual deflection should be 20% less. With live load in the half span, the deflection $\delta_v$ at the three-tenth point caused by the shortening of the arch is equal to $\delta_v = 0.7 \times 0.55 = 0.39$ in., approximately. The corresponding dead-load deflection $\delta_v$ caused by the shortening of the arch is equal to $\delta_v = 0.7 \times 4.2 = 2.9$ in., approximately; and the temperature deflection, $\delta_v = 4.19$ in., or a total of about 14.48 in. Although, possibly, these deflections may be reduced 20% on account of the increased stiffness of the rib toward the springing, they are nearly twice as large as those assumed by the authors on the basis of the deflection theory.

A particularly valuable part of this Symposium is the inclusion of Table 1, which enabled the writer to compare the results of the "elastic theory," as propounded by M. Bresse, with the "deflection theory," defended by the Symposium authors.

O. H. AMMANN,[37] M. AM. SOC. C. E.[37a]—The designers and constructors of the Rainbow Arch Bridge deserve credit for having produced a most excellent structure, both esthetically and technically. The arch type of bridge, like its predecessor, undoubtedly suggested itself as the natural solution because

[37] Cons. Engr., New York, N. Y.

[37a] Received the Secretary January 24, 1944.

of the steep rocky sides of the river gorge. The banks are of sufficient height to permit the bridge floor to be entirely above the arch ribs—an arrangement which is esthetically most ideal for any arch structure.

The appearance is enhanced by utmost simplicity of the structural design, more particularly the selection of slender solid-plate arch ribs in place of deep arch trusses which have heretofore prevailed in arches of this magnitude. The adoption of solid-plate arch ribs in this bridge was undoubtedly made possible and economical by the recent advances in theory and structural design as well as in methods of fabrication and erection, and, therefore, evidences a further milestone in the art of bridge construction.

The design of the graceful imposing arch span proper is kept strictly to structural necessities, free from ornamental or architectural embellishments. The short approaches surmounting the rocky slopes are appropriately designed to appear as massive arch viaducts; their contribution to the monumental appearance of the structure undoubtedly justified the additional expense involved when compared to light steel structures exemplified in some similar cases.

Under the given conditions the adoption of the hingeless type of arch rib was unquestionably most appropriate. A two-hinged arch of satisfactory performance could have been designed but, as is indicated by the authors' comparative figures, such an arch with slender solid-plate ribs would have been considerably less economical than the hingeless type.

The arguments might be advanced that the two-hinged arch is statically indeterminate in a lesser degree and that yielding of the foundations, or inaccuracies in workmanship and erection, especially at the rib splices, might cause more serious secondary or supplementary stresses in the hingeless arch. Neither of these arguments could be of practical importance in this case. The determination of stresses, even in complex statically indeterminate structures, has advanced to a stage which, for an important structure, does not admit greater simplicity of analysis as a valid argument. Provided there is no external yielding, the stresses in any well-designed and well-built statically indeterminate structure can be determined with the same degree of reliability as those in a statically determinate system. As regards the effects of yielding, the solid rock foundations, and the high degree of accuracy of fabrication and erection which is now attained in steelwork of this nature by experienced and well-equipped constructors, preclude supplementary stresses of practical significance.

In view of the slenderness of the arch ribs, which was undoubtedly dictated by the practicable make-up of the solid-plate section, the deformations under load could be expected to be appreciable and to warrant investigation of the stresses by a more accurate analysis than the so-called "elastic theory," more particularly since in an arch the differential stresses from the deformation are additive.

The designers have made such an investigation, at least for certain points, and have developed an analysis and approximate formulas for more accurate stresses which, as far as they have been applied, appear to be adequate for

this particular structure. It seems to the writer, however, that the permissible limits should be assigned to the accurate stresses and not to the ones determined by the inadequate "elastic method." It is true that in this particular case the more accurate stresses, as far as calculated by the designers, are still within safe limits, but it would seem that, in general, and particularly in comparative designs based on the same specifications, the prescribed permissible stresses should apply uniformly to stresses determined by an adequately accurate method. The differential stresses cannot be classified as "secondary" stresses in the commonly accepted sense.

In this connection, the writer has long held the view that the term "elastic theory," by which the analysis of stresses in an assumed rigid system has become known, is incongruous and misleading. If a special term for it appears desirable it should be the "theory of the rigid system"; and what has become known and referred to in the first paper of the Symposium as the "deflection theory" should be termed, in distinction, "theory of the flexible system." Fundamentally, there is no distinction between the two, except the degree of accuracy, both being based upon the laws or the hypothesis of elasticity. For most common structures in which the changes in the form of structure under loads are negligible, the theory of the rigid system is sufficiently adequate. For more flexible structures there are different "theories of the flexible system" depending upon the degree of accuracy desired or justified. Any desired degree of accuracy may be attained by a method of trial and correction, by more or less elaborate cut and dried formulas, or by model tests. The authors, very wisely, have resorted to all three methods in an effort to check the results, at least at certain critical points.

The research work on two-hinged arch models described in the second paper of the Symposium, and the method of analysis derived therefrom, should prove of aid to the designer, especially in connection with rough preliminary studies. It is the kind of research that deserves to be encouraged in connection with the design of any unusual structure. The results obtained demonstrate again the reliability of stress determinations by means of models and their usefulness in checking the adequacy of theoretical analysis.

It is only regrettable that circumstances probably did not permit the work to be extended to include a complete check of stresses on a model representing the hingeless arch of the Rainbow Bridge as actually built, and it is to be hoped that opportunity for such a study will present itself in the future.

The writer cannot agree with the conclusion expressed by the authors of the second paper that two-hinged arches should be of sufficient stiffness to make the application of the "elastic theory" entirely proper. As long as the accurately determined stresses from the maximum specified loads are within the prescribed limits the design should be considered satisfactory. Accepted practice does not determine the design load by applying a uniform factor of safety to a hypothetical yield load, but rather limits the accurate unit stresses under maximum design load to certain safe stresses based upon the qualities (especially the yield point) of the material.

The third and fourth papers of the Symposium reflect the elaborate and

thorough studies and preparations made by the constructors to insure accuracy in workmanship and perfect control of the stress conditions during erection.

Corrections for *Transactions:* In October, 1943, *Proceedings*, Fig. 6, page 1172, change span dimension to "23 Panels @ 39′ $4\frac{3}{8}''$ = 905′ $4\frac{5}{8}''''$"; see also, footnote [16b], page 1215; in Fig. 10, page 1184, show a tangential displacement $v$ of point $a$; change Eqs. 3, page 1184, to read: "$\eta = u \cos\theta - v \sin\theta$; and, $\rho = u \sin\theta + v \cos\theta$; in which $v$ is the tangential displacement. In the usual * * *"; in Figs. 16, 17 and 19, change $W$ in all cases, to $\overline{DL}$; in Eq. 16, page 1187, change "$M_{ae}$" to "$u_{ae}$"; on page 1197, delete the sentence beginning on line 8: "The apparatus * * * the fixed-end moments"; and, on page 1199, line 10 following Eq. 23*b*, change "Master of Science" to "Civil Engineer."

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DISCUSSIONS

# RIGIDITY AND AERODYNAMIC STABILITY OF SUSPENSION BRIDGES

## Discussion

BY EDWARD ADAMS RICHARDSON

EDWARD ADAMS RICHARDSON,[7] ESQ.[7a]—Although this discussion must be limited primarily to aerodynamical considerations, the calculation of vibrating systems deserves comment. For example, the work of H. Reissner,[8] on modes of vibration, should be cited. Mr. Reissner represents the period of oscillation of the cable alone ($P_{cn}$ in seconds) by the equation

$$P_{cn} = \left(\frac{1}{n}\right) C\ 0.9968\ \sqrt{f} \qquad (67a)$$

in which (in addition to the notation of the paper), $C$ is a coefficient, depending on the sag ratio $\left(\frac{f}{l}\right)$, not greatly different from 1.0. With $C = 1.0$, this equation can be derived from that given in the *Encyclopædia Britannica*[9] for taut strings; hence it applies to any flexible suspended cable, string, or chain of uniform weight. Theodor von Kármán, M. Am. Soc. C. E., and M. A. Biot[10] have shown that the period of vibration $P_{gn}$ of a uniform beam, uniformly loaded, supported at the ends, may be represented by the equation

$$P_{gn} = \left(\frac{1}{n}\right)^2 \left(\frac{2}{\pi}\right) l^2 \sqrt{\frac{w/g}{E\,I}} \qquad (67b)$$

in which $w$ should be the dead load applied to one girder when $I$ is the moment of inertia of that girder, excluding the corresponding weight of the cable and suspenders.

The combined period $P_{Bn}$ of cable and its corresponding girder (in seconds) may be found from the fundamental cable period $P_{c1}$, the fundamental girder

NOTE.—This paper by D. B. Steinman, M. Am. Soc. C. E., was published in November, 1943, *Proceedings*.

[7] Special Engr., Bethlehem Steel Co.; Partner, Edward, Nellie and George Richardson, Bethlehem, Pa.

[7a] Received by the Secretary January 11, 1944.

[8] "Oscillations of Suspension Bridges," by Hans Reissner, *Journal of Applied Mechanics*, A.S.M.E., March, 1943, p. A-23.

[9] *Encyclopædia Britannica*, 14th Ed.

[10] "Mathematical Methods in Engineering," by Theodor von Kármán and M. A. Biot, McGraw-Hill Book Co., Inc., New York and London, 1940.

or truss period $P_{g1}$, and the ratio $q = \frac{P_{c1}}{P_{g1}}$ through use of the equation

$$P_{Bn} = \frac{P_{c1}}{n + (n\,q)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (67c)$$

This simple method of combining cable and either the girder or truss periods, to obtain a combined period equal to that of the bridge for vertical oscillations, yields substantially the same results as Eqs. 1 and 6. It is sufficiently accurate for bridges of interest from the standpoint of aerodynamic instability, as Mr. Steinman suggests. Torsional periods may be derived from Eqs. 67 by multiplying by a coefficient[11] of the form $\frac{2\,r}{b}$ provided it is remembered that the weight of cables and suspenders forms part of the span cross section whose polar radius of gyration is to be used.

It should be emphasized that the author's *K*-limits for flexible bridges, constructed in a similar manner and of similar materials (such that the several cable and girder systems are substantially independent) should not be considered to apply to structures departing materially from such type similarity in any important respect.

*Part 3.—Aerodynamic Excitation of Torsional Oscillations.*—It is regrettable that this paper contains so few references to the theoretical and experimental work that has been done in the field of aerodynamics.

For example, the paper is based on the alleged necessity that both lift and moment curves of the cross sections of bridges shall have regions of negative characteristic trend. Yet such curves probably never apply to actual bridges, and it can be shown by the work noted in the *Encyclopædia Britannica*[9] on wires made to vibrate by uniform wind streams, by the work of D. Thoma[12] on the lateral vibrations of circular cylinders in uniform streams of water when the spring factor and damping were varied, as well as by papers of other writers, that no such necessity exists. Furthermore, the tests,[11] on which this characteristic was found, were made on a model of low aspect ratio (short compared with the model width) and do not agree with those made at the Guggenheim Aerodynamic Laboratory at Pasadena, Calif., on models of effectively higher aspect ratios (which tests were given in the same report).[11]

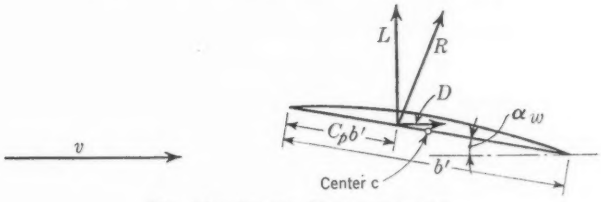


FIG. 10.—AIRFOIL CHARACTERISTICS

[11] "The Failure of the Tacoma Narrows Bridge," a report to the Hon. John M. Carmody, Administrator, Federal Works Agency (see brief résumé in December, 1943, *Proceedings*, p. 1564).

[12] "Flatterschwingungen von Zylindern in gleichmäszigen Flüssigkeitsstrom," by August Meier-Windhorst, *Mitteilungen des Hydraulischen Instituts der Technischen Hochschule*, Heft 9, München, 1939, p. 1.

Referring to Fig. 10, an airfoil having a chord $b'$ is set at an angle of attack $\alpha_w$ to the direction of the uniform wind stream which has a relative velocity $v$ remote from the airfoil. The area of the airfoil projected on the plane of the chord and span is $A_{ls}$. The resultant force $R$ crosses the chord at a distance $C_p\, b'$ from the leading edge. The vertical component of $R$ is the lifting force $L$, and the horizontal component is the drag force $D$. As used in Fig. 10, $q$ represents the pitot-tube pressure (impact pressure) of the air stream in pounds per square foot. Appropriate formulas for this case are: Impact pressure—

$$q = \tfrac{1}{2}\,\rho\, v^2 \qquad (68a)$$

lift coefficient—

$$C_L = \frac{L}{q\, A_{ls}} \qquad (68b)$$

drag coefficient—

$$C_D = \frac{D}{q\, A_{ls}} \qquad (68c)$$

and, moment coefficient (any axis)—

$$C_M = \frac{M}{q\, A_{ls}\, b'} \qquad (68d)$$

For bridge oscillations, the moment coefficient $C_{Mc}$ about the center of the chord is desired, or, if found by the formulas—

$$r = C_L \cos \alpha_w + C_D \sin \alpha_w \qquad (69a)$$

and

$$C_{Mc} = r\,(0.50 - C_p) \qquad (69b)$$

—will approximate the desired value.

When the airfoil shown in Fig. 11 suddenly acquires a velocity $v$ relative to the air at rest (or, when the air "at infinity" has the velocity $v$ relative to the airfoil), a vortex is formed which travels away from the airfoil at a velocity less than $v$, swirling in the direction shown. Any change in relative velocity will cause the generation of a vortex of appropriate magnitude and sense. In Fig. 11, any sudden stoppage would set up a vortex equal in magnitude to

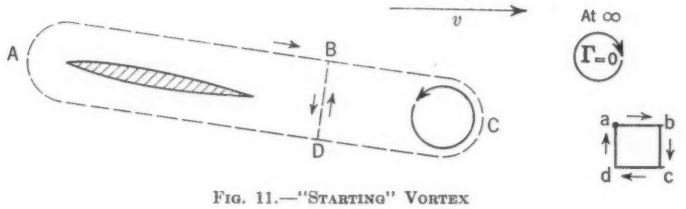


FIG. 11.—"STARTING" VORTEX

that of the starting vortex, but of opposite rotation. As long as the airfoil maintains its velocity at a constant angle $\alpha_w$, it will be found that the circulation remains constant around the airfoil.

Circulation $\Gamma_x$ may be assumed constant for an element of wing with a length $dx$ along the span, in the plane, at $x$ feet from some spanwise reference

point, perpendicular to the span and parallel to the air stream. In vector notation,

$$\Gamma = \int \bar{v}\, d\bar{s} \qquad (70)$$

in which $\bar{v}$ is the vector point velocity on the path of integration, and $d\bar{s}$ is the vector element of the path of integration at the same point. Thus, circulation has the dimensions $\frac{\text{ft}^2}{\text{sec}}$.

Remote from the airfoil in a wind stream of uniform velocity $v$ free from turbulence, from Fig. 10 and the square abcd the circulation may be seen to be zero. Since $\bar{v}$ is normal to $\overline{da}$ and $\overline{bc}$, the integral along these sides of the square is zero. Since $\bar{v}$ is the same for $\overline{ab}$ and $\overline{cd}$, whereas $\overline{cd} = -\overline{ab}$, the sum along these two sides is zero. Hence, for a uniform wind stream the circulation must be zero in any part of the wind stream where the path of circulation does not cut a vortex system.

Referring again to the starting vortex (Fig. 11), the path ABCD encloses the entire system of wing and starting vortices, and is remote from both, so the circulation along it is zero; but along the path BCD the circulation must correspond to the strength of the starting vortex. Hence, along the path ABD, the circulation must equal in magnitude that of the starting vortex, but be of opposite sense so that the sum may be the zero required for the path ABCD.

The theorem of Kutta-Joukowski requires that the lift of an airfoil element of span $db''$ shall have the value[13]

$$dL = \rho\, v\, \Gamma_x\, db'' \qquad (71)$$

For a finite airfoil with free wing tips, the lift distribution spanwise may be represented by the ordinates of a semiellipse whose major axis is the span $b''$ and whose semiminor axis is $\Gamma_0$ at $x = 0$. The value of $L$ takes the form of $\rho\, v$ times the integral of $\Gamma_x\, db''$ which in turn may be taken as $\Gamma_w\, b''$. Thus, any change in lift may be due either to a change in $v$, a change in $\Gamma_w \left( = \frac{\pi}{4}\, \Gamma_0 \right)$, or a combination of each.

Fig. 12 shows a rectangular wing with a chord $b'$, and a span $b''$, in an air stream with a velocity $v$ remote from the airfoil and directed normal to the span. The wind area $A_{ls}$ equals $b'\, b''$. The total wing circulation producing lift is shown by the curve $\Gamma_w\, b''$, which is defined by:

$$\Gamma_w\, b'' = \frac{L}{\rho\, v} = 2 \int_0^{b''/2} \Gamma_x\, dx \qquad (72)$$

A tip vortex is being generated continuously. This is true only for the finite wing, since a wing of infinite span or one with shrouded tips would have no tip vortices. Work is required to generate these vortices, and a part $D_i$ of the drag

[13] "Applied Wing Theory," by Elliot G. Reid, McGraw-Hill Book Co., Inc., New York and London, 1932, Chapters III and IV.

$D$ furnishes the resistance required, or the induced drag. The remainder of the drag is the profile drag $D_p$, which is equal to $D - D_i$. The value of $D_i$ may be found from the equation[13]

$$D_i = \frac{L^2}{\pi\, q\, (b'')^2} \qquad (73)$$

Eq. 73 is used in converting the data for a wing or wing model of one aspect ratio to those for some other aspect ratio. It would not apply very consistently to the models of the Tacoma Narrows Bridge due to the action of the

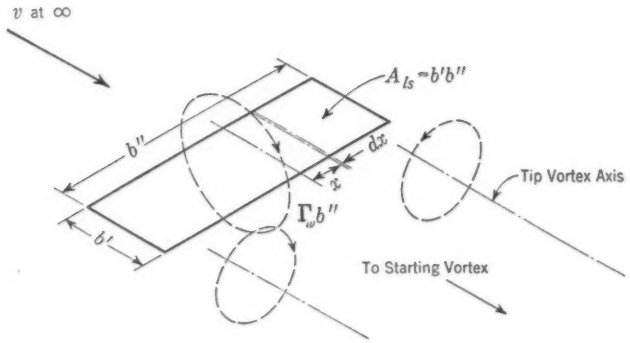


FIG. 12.—AIR FLOW FOR FINITE WING

longitudinal girders. The effect of the tip vortices is to produce a descending velocity component in the wake of the airfoil. For infinite span length, Eq. 73 becomes zero.

Assume a model (Fig. 13), for which the angle of attack is zero in a uniform wind stream of velocity $v$ remote from the airfoil. Assume also that the entire drag $D$ is now the profile drag $D_p$ due to the friction of the air stream on the

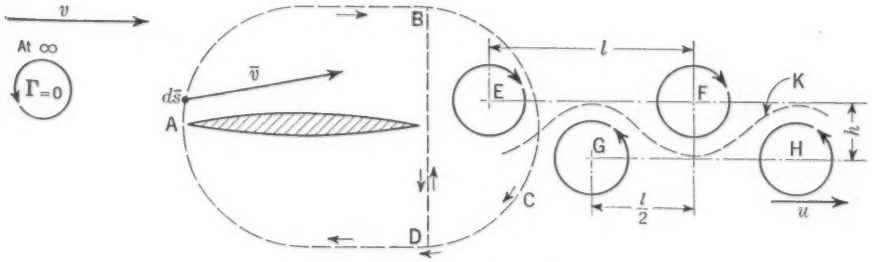


FIG. 13.—THE VON KÁRMÁN TRAIL

airfoil. Above the very lowest values of $v$, the trailing air stream contains vortex elements E and F whirling in one direction, alternating with others, as G and H, whirling in the opposite direction. The velocity $u$ in the vortex system is less than $v$, relative to the airfoil. For a steady system of this type,[14]

[14] "Applied Hydro- and Aeromechanics," based on lectures of L. Prandtl, by O. Tietjens, McGraw-Hill Book Co., Inc., New York, N. Y., 1934, p. 130 et seq.

$\frac{h}{l} = 0.2803$. If $l$ is known for the vortex period $P_K$, the drag, and the circulation of each vortex emitted, may be computed very closely. Knowing the circulation for the vortex E, for example (and it could be measured along a path BCD, Fig. 13), the circulation along the path ABD about the airfoil section must be of about the same magnitude as that for E but opposite in direction, since the circulation along ABCD is almost zero. The other vortex elements (F, G, and H) have slight influence. Hence the circulation about the airfoil section must vary with the same frequency as that for vortices E to F from $+\overline{E}$ to $-\overline{E}$. By the Kutta-Joukowski theorem, the lift $L$ normal to the chord must vary correspondingly. Hence a periodic lateral force acts on any "lifting surface" (even a circular cylinder) when placed in a uniform air stream and this force acts without any reference to the airfoil characteristics of lift.

It is convenient to use the parameter $\eta$ introduced by Mr. Thoma to describe the behavior of airfoils, in which

$$\eta = \frac{b' P_K}{v} \qquad (74)$$

Other writers[15] have used the parameter $\left(\frac{1}{k}\right)$ which is the same as $\frac{\eta}{\pi}$. In terms of this parameter, Mr. Thoma[12] found that $\eta = 5.0$ for a stationary circular cylinder. With low damping, oscillation of cylinders occurred at values as low as 4.73, initially, with maximum oscillations at values greater than 6.00; but amplitudes decreased rapidly, toward the end of the critical region, to about 6.8. The *Encyclopædia Britannica*[9] used 5.41 for the vibration of wires in a wind stream. In the case of bridge floors, it is probable that a value of 6.0 would be approximately correct. Tests on wing flutter,[16] and on the oscillation of bridge models,[11] all show the effect.

In other words when $P_K$, with $\eta = 6.0$, is the same as $P_{Bn}$, large oscillations may occur, and the corresponding optimum value of $v$ may be found for the particular mode of oscillation. However, since the characteristic dimension, $b'$, of an airfoil is measured parallel to the air flow (and therefore is correct only for wind directions normal to the span), the corresponding dimension for a wind making the angle $\gamma$ with that normal is $b'$ sec $\gamma$. Thus the wind velocity must be increased in direct ratio to sec $\gamma$ to keep the von Kármán trail period equal to that of the bridge. This is equivalent to retaining $b'$ as the characteristic dimension, using with it only the normal velocity component of the wind, $v$ cos $\gamma$. As the energy of the wind increases with $v$, it is probable that the greater $\gamma$ becomes, the more certain the wind will be in starting oscillations. For example, the tests of Louis G. Dunn[11] showed that the Tacoma Narrows Bridge would have negative aerodynamic damping, with torsion at $n = 2$ for values of $v$ greater than 25 miles per hr. With $\eta = 6.00$, a value of $v$ of 27

[15] "General Theory of Aerodynamic Instability and the Mechanism of Flutter," by Theodore Theodorsen, *Report No. 496*, National Advisory Committee for Aeronautics, 1935.

[16] "A Study of Wing Flutter," by A. F. Zahm and R. M. Bear, *Report No. 285*, National Advisory Committee for Aeronautics, 1928.

miles per hr would be needed. At failure, the wind velocity was 42 miles per hr, with $\gamma = 50°$, so that the normal component was 27 miles per hr. The agreement in this case is excellent. Mr. Steinman's computation requires 42 miles per hr, but he fails to mention the influence of wind direction and the wide difference between his results and the model tests made by Mr. Dunn. The von Kármán trail seems adequate to account for the motion of the bridge. It provides a method of approximating wind velocities at which failure, or serious vibrations, may occur, but no one can be sure that they will actually occur.

It is possible[14] to express the equation for profile drag in the form

$$D_p = 0.959\left(\frac{l}{b'}\right) - \left(\frac{0.331}{\eta}\right)\left(\frac{l}{b'}\right)^2 - \left(\frac{0.628}{\eta^2}\right)\left(\frac{l}{b'}\right)^3 \quad \ldots\ldots (75)$$

which indicates that, at constant drag, $l$ and $\eta$ may take various values, with a tendency for the trail period to be unsteady. If, instead of a uniform approaching air stream, the air stream is turbulent, with a circulation changing irregularly in period and intensity, there are two factors tending to combine in producing the lifting surface circulation. It is quite possible that a massive structure (or one that is so stiff that the oscillation per cycle would be small compared with the range of disturbances in the circulation; or, one with large damping effect) would show no signs of oscillation at the critical velocity of air stream. Given a reasonably small starting oscillation, however, it would proceed to increase in amplitude with continued wind action because, when the surface does oscillate under the governing control of spring frequency, periodic "starting" vortices are superposed on the von Kármán trail tendency and the trail unsteadiness tends to disappear, favoring increasing oscillation amplitudes.[17]

If a bridge is so light or flexible that it tends to undergo forced oscillations of importance, even relatively small degrees of von Kármán trail or of turbulence in the air stream may suffice to set up important amplitudes of oscillation. This was true of the Tacoma Narrows Bridge which revealed many modes of oscillation when the wind velocity exceeded 5 miles per hr. Other bridges have shown trains of waves crossing the span, possibly set up by a short gust at some point of the bridge length (not a gust enveloping the entire structure), as in the case[11] of the Golden Gate Bridge.

In passing, it is to be noted that the action of wind on towers may be important because the towers generally have $b'$-values quite different from bridge floors.[18]

The low power input at very small amplitudes, and the rapid rise in power input as the amplitude increases, were demonstrated well by Mr. Dunn.[11] One example is given in Table 7. Since the kinetic energy varies as the square of the amplitude, the gain in energy is about 3.5 times as great in each second as in the preceding second. Many tests showed no certain starting from rest

[17] "Suspension Bridge Oscillations and Failure," by Edward Adams Richardson, a report to Advisory Board on Suspension Bridges, Bureau of Public Roads, Federal Works Agency, November 13, 1943 (revised).

[18] Reports to the Advisory Committee of the Bureau of Public Roads, Federal Works Agency, on Long Span Suspension Bridges, by Sterling Johnston, January 15 and May 25, 1943 (unpublished letters).

in a uniform wind stream, but did show this form of increase in amplitude when given small initial amplitudes.[17] The tests suggested strongly that turbulence has a marked effect. It should be remembered that in the case of the Tacoma Narrows Bridge there was very little damping in either the models or the prototype.

TABLE 7.—VARIATION OF POWER INPUT WITH AMPLITUDE

| Time (sec) (1) | Amplitude (degrees) (2) | Col. 2 squared (3) | Gain in Col. 3 (4) |
|---|---|---|---|
| 0 | 0.67 | 0.45 | |
| | | | 1.11 |
| 1 | 1.25 | 1.56 | |
| | | | 3.78 |
| 2 | 2.32 | 5.34 | |
| | | | 11.88 |
| 3 | 4.15 | 17.22 | |
| | | | 46.78 |
| 4 | 8.00 | 64.00 | |

Mr. Steinman has given equations for bridge damping that are not general, since material, type of construction (such as welding or riveting), and the introduction of damping devices may change structural behavior greatly. To reduce, or prevent, oscillations, high initial damping or friction is essential. Damping should be sufficiently high to limit oscillations if sudden impulses set up amplitudes which might otherwise lead to wider swings. For example, the dead load of the bridge floor and supports might be transferred to the trusses or girders through friction shoes resting on friction slides attached to these trusses or girders, so that a slight change in relative position, as on changes in deflection, would be opposed by the high starting friction of such sliding devices. Such an arrangement might be sufficient to prevent oscillations, escaping from the irregular turbulence range of forced oscillations, from growing, when the general air-stream speed is synchronized with some particular mode of oscillation.

Oscillating airfoils have been studied extensively and, contrary to the author, so have the phase relations presented in the paper. It is unfortunate that Mr. Steinman's studies did not include the von Kármán trail effect, as that concept is essential to a workable theory of the starting of oscillations from rest by a wind stream. Tests under conditions appropriate to these studies and theories, in which the von Kármán effect was small, checked the theories quite well.[12, 15, 19, 20, 21]

It is essential that the theory of probability be considered in determining the possibility of oscillations being started by the wind stream. Resonance in general requires that, to produce its maximum effect, the oscillating force shall have a frequency varying, within narrow limits, from that of a given possible bridge frequency. The wind speed at such theoretical synchronism is calculable, as has been noted; but the trail frequency itself tends to be irregular, and for heavy and stiff bridges the force per cycle is capable of producing only a very small oscillation amplitude. Only at appreciable amplitudes will the governing effect of the bridge frequency, as an oscillating airfoil, produce trail

[19] "Force and Moment on an Oscillating Airfoil," by H. Glauert, *Reports and Memoranda No. 1242*, British Aeronautical Research Committee, 1929.

[20] "On Some Reciprocal Relations in the Theory of Nonstationary Flows," by I. E. Garrick, *Report No. 629*, National Advisory Committee for Aeronautics, 1938.

[21] "Experimental Verification of the Theory of Oscillating Airfoils," by Abe Silverstein and Upshur T. Joyner, *Report No. 673*, National Advisory Committee for Aeronautics, 1939.

frequency regularity. Hence, a light and flexible bridge, which can be given an appreciable amplitude with a short train of impulses that happen to be quite close to the frequency required, is much more likely to begin vibrating than a stiffer and heavier bridge for which the probability of a sufficiently long impulse train with proper frequency is small. The succeeding, out-of-frequency wave trains tend to dampen the slight motion started. If, in addition to the irregularities of the von Kármán trail, turbulence irregularities of the wind stream itself[22] are added (which there is no reason to expect will be of the proper frequency), the chance of producing oscillations greater than the irregular trail and turbulence effects is slight. Hence the Ammann criterion[23] separates the usual types of bridges and bridge construction into two classes (oscillating bridges and non-oscillating bridges) much more sharply than would appear reasonable. However, because a bridge has not oscillated or failed in a wind stream of critical velocity cannot be considered a guarantee that it may not fail, since turbulence varies considerably with conditions; and it may be possible for vibrations to start with reduced turbulence. It is of the utmost importance that von Kármán trails be studied further with respect to forces applied and steadiness. At the same time, turbulence at the bridge should be studied, for various wind velocities and directions at different times, so as to evaluate possible ranges and the relation of such phenomena to the magnitude of the von Kármán trail.[17]

In any case where oscillation can start, the limiting amplitude will be that at which the work done by the given wind on the bridge is just dissipated in damping. Of two winds with the same normal component, the one having the higher velocity will produce the larger oscillations, since the forces are larger. The tangential component has been neglected herein, but it cannot be ignored.

In general, vertical oscillations in phase with the wind will not be free from small torsional oscillations of the forced vibration kind, and vice versa.

*Part 4.—Methods of Securing Rigidity and Aerodynamic Stability.*—The fact that rigidity, of the kind discussed by Mr. Ammann,[23] is desirable needs no discussion. The statement by the author that $R$, the proportion of rigidity contributed by the stiffening structure, should be large, because of the accompanying increase in structural damping, is true enough and was well demonstrated[17] by the behavior of the Lion's Gate Bridge at Vancouver, B. C. ($R$ being about 0.32).

Although the Ammann type of rigidity is desirable, it is not essential, since the limits set assume substantially independent vibration by the two cables normally used. The transverse diagonal stays recommended by the author are intended primarily to prevent torsional oscillations and thereby the possibility of torsional failure. The action is perhaps somewhat different from that described by Mr. Steinman. If displacement of one cable relative to the other begins, anything that reduces the load on the descending cable and transfers it to the other cable will tend to keep the cables in phase. This

[22] "Turbulence Factors of N. A. C. A. Wind Tunnels as Determined by Sphere Tests," by Robert C. Platt, *Report No. 558*, National Advisory Committee for Aeronautics, 1936.

[23] Report of March 15, 1943, by O. H. Ammann to the Subcommittee on Model Studies, Advisory Committee on the Investigation of Long Span Suspension Bridges, Bureau of Public Roads, Federal Public Works Agency.

effect of the interchange of load on stability was observed by A. F. Zahm and R. M. Bear.[16] Since a diagonal consisting of one half the suspenders will be fully loaded with a deflection of the order of 1/10,000th of the possible amplitude of oscillation, the stiffening effect of such diagonals is extremely great. Oscillations are prevented before they can start, and normal torsional moments are handled easily. No increase in suspender size is needed when half the suspenders are in the diagonals, for the action is similar to that of the lacing bars of a lattice column. Furthermore, since vertical oscillations cannot be totally independent of torsional ones (small and forced), such diagonals will reduce but not prevent vertical oscillations—that is, unless the amount of reduction limits the possible oscillations to the order of those forced by the irregular turbulence.

Obviously, the longitudinal diagonal stays of the intermediate type act somewhat similarly to the transverse diagonals, since two floor points are connected to one cable point. If one floor point tends to move downward, its load is transferred to a cable point farther along the span and tends to cause that point to follow suit. Hence, such a system of stays may very well act to prevent vertical oscillations. The two systems together should make a very flexible bridge act as though it were much more rigid. The additional weight involved is relatively small.

Clearance considerations might limit the use of diagonal stays to the end portions of the bridge. In that case[17] diagonal stays in the center section, from a cable across the adjacent walkway to the edge of the floor, could be added for measurable control. The center walkway is thus forced outside of the girders; but since bridge oscillations depend on the floor width, $b'$, the use of one width in the center and another at the ends would in itself operate to limit oscillations. The two sections (having different von Kármán trail periods) would not be in synchronism at the same time.

The author's description of the outer floor construction and of open roadways is not clear. For instance, the statement (side heading, Part 4, "Use of Open-Floor Construction"), "The solid middle strip of roadway contributes to aerodynamic stability by atmospheric damping action" ignores the fact that no such effect occurs when most needed; that is, when the bridge and trail periods are in synchronism.

As noted, center stays are of value as long as they last; but localized control leads to large forces, whereas distributed control, as by the use of the two types of diagonal stays, permits no such forces to develop as may destroy the center-stay system.

Although differing materially from the author on many important points, the writer tends to agree with him in his final conclusion and might add that even much more slender bridges than Tacoma Narrows could be made to behave as if they were much stiffer, at the same time being both comfortable and safe.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# UNUSUAL CUTOFF PROBLEMS—DAMS OF THE TENNESSEE VALLEY AUTHORITY

## A SYMPOSIUM

### Discussion

BY BERLEN C. MONEYMAKER

BERLEN C. MONEYMAKER,[4] Assoc. M. Am. Soc. C. E.[4a]—An interesting and valuable record of how a serious and difficult foundation problem was solved successfully is presented in Mr. Hays' paper in this Symposium. Solution channels create foundation problems at nearly all limestone dam sites, but the large channel at Kentucky Dam presented a problem of unusual magnitude, involving both bearing strength and watertightness. So far as is known, this channel is the largest solution cavity ever to be encountered and treated in any part of a dam site. It lies entirely below the water table and is from 70 ft to 100 ft wide at the top (El. 289), decreasing in width with depth to El. 70±, where it disappears. The linear extent of the channel is unknown, but it is more than 1,000 ft long, and there is good evidence that it is more than 2,000 ft long. The channel extends across the lock and dam in the direction of N 02° W, but its trend changes to N 07° W a short distance downstream.

Immediately downstream from the lower lock gate, where the cutoff was effected by a subterranean arch dam 195 ft high, built from the top downward, the channel is 70 ft wide at the top. It extends downward without any appreciable decrease in width to El. 217, where it bifurcates, and from here the two prongs taper to extinction at about El. 70. The channel was nearly completely filled with residual chert and clay, sand, and blocks of limestone. In the upper region of the channel the limestone blocks are flat-lying and appear to be but little out of place. Although the channel was formed by the solution of the rock, it was never a large open cavity. Solution progressed along numerous closely spaced parallel joints, producing a series of vertical channels which subsequently were filled by material which slumped from the sides and from above. Solution, followed by cavity filling, progressed until nearly all of the rock in place was dissolved.

NOTE.—This Symposium was published in November, 1943, *Proceedings*.

[4] Chf. Geologist, Geologic Div., TVA, Knoxville, Tenn.

[4a] Received by the Secretary December 23, 1943.

The deep solution channel was not found in the early stages of exploration; in fact, it was discovered little in advance of the actual excavation of the rock in the area involved. Although the Geologic Division of the TVA did not conduct the exploration activities at Kentucky Dam, it is appreciated that it was not practicable, however desirable it might have been, to drill closely spaced holes to within 100 ft of sea level over the entire dam site. As the dam alone is about 8,700 ft long, the total area under exploration was more than 10,000 ft long and several hundred feet wide, and ranged in elevation from 300 ft to nearly 450 ft. With the exception of two small outcrops, bedrock was covered by an overburden of alluvial and residual materials which varied in thickness from 20 ft to 110 ft. In view of the thickness and continuity of the overburden, the necessity of locating the exploration holes on fairly wide centers, the relatively narrow and linear character of the channel, and the presence of large and little disturbed blocks of limestone in the cavity filling down to El. 230, it is not surprising that the channel was not discovered in the course of the preliminary exploration.

The exploratory drilling revealed that, in places, the rock in the lock area was extensively dissolved, that the solution cavities were predominantly vertical channels, and that some of the cavities extended to great depths. Prior to January, 1939, only two holes, both relatively shallow, had been drilled into the channel. Both holes encountered numerous filled solution cavities alternating with layers (blocks) of undissolved limestone. As the same condition prevailed in narrow vertical solution channels, the vertical drill holes afforded no clue as to whether the solution channel was 1 ft wide or 100 ft wide. However, as additional holes subsequently were drilled, it became evident that rock was intensely and deeply dissolved in certain areas. Deep holes drilled into one of these areas at various angles disclosed that the deep and intense solution was confined mainly to a well-defined zone. Intensive drilling soon revealed the linear character of the zone, its width and its depth, and the extent of solution within the zone; but the details of geology as shown in the geologic section (Fig. 9) were not fully appreciated until excavation for the cutoff was undertaken.

Precisely how the cutoff of this large channel was effected is very clearly and concisely reported in Mr. Hays' paper. He is to be congratulated upon contributing to the engineering profession such an excellent record of the problem and its solution.

Mr. Schmidt deals with another unique cutoff problem. The excessive leakage at Hales Bar Dam has been referred to so frequently in engineering and geological publications that it has been accepted as the classic example of a dam built with insufficient foundation investigation, preparation, and remedial treatment.

When Hales Bar Dam was built (1905–1913), the science of engineering geology had not been developed. At that time, large masonry dams were few in number, were widely separated geographically, and were built with little or no geologic investigation. Even until 1933 most large masonry dams in the United States were built without more geologic investigation than a few cursory examinations by one or more consulting geologists. Until 1930 American geologists were almost unanimously of the opinion that solution cavities could not develop in limestones lying below the water table. It is

not surprising, therefore, that Hales Bar Dam was built without an adequate geological investigation. Moreover, it is extremely doubtful whether a mere geologic investigation of the type current from 1905 to 1913, without extensive exploration with core drills, would have revealed in advance the serious foundation defects which caused trouble and delays during construction and appreciable leakage after completion of the project.

Hales Bar Dam is located on the Bangor (unrestricted) limestone of Mississippian age. The formation consists mainly of limestones with which are interstratified shaly layers of various thicknesses. The limestones, in the main, are quite pure, containing as much as 99% calcium carbonate. Structurally, the rock is almost horizontal and is cut by numerous joints and a few minor faults.

As stated by Mr. Schmidt, much of the rock under Hales Bar Dam is cavernous. The solution cavities, which are quite numerous in certain areas have developed mainly along joints, faults, and bedding planes. Some of them are large and open, and they are present down to El. 480, or more than 100 ft below the original river channel.

In effecting a cutoff under the dam, intersecting 18-in. holes had to be drilled and concreted to depths as great as 100 ft. The cavernous character of the rock rendered the task difficult at best, but the difficulties were enhanced appreciably by large volumes of water under considerable head, moving through the cavities at high velocities.

Mr. Schmidt's paper is timely and is of considerable interest to both engineers and geologists. In view of the widespread interest in the leakage problem at Hales Bar Dam, the paper is a valuable report on an entirely new method of cutting off excessive leakage.

In both papers of this Symposium, the value of detailed geologic work in the solution of the problems at hand is obvious although perhaps not sufficiently emphasized. At the Kentucky project the geologic work was done, successively, by Roger F. Rhoades, Portland P. Fox, Assoc. M. Am. Soc. C. E., and Leland F. Grant. The geologic studies involving the deep solution channel, discussed by Mr. Hays, were made first by Mr. Rhoades and later by Mr. Fox. At Hales Bar Dam the work was done first by Cecil B. McGavock, Jr., and later by John W. Frink, Juniors, Am. Soc. C. E. The geologic exhibits used in the two papers were adapted from drawings prepared by these men.

Corrections for *Transactions:* In November, 1943, *Proceedings:* Fig. 13, at Stations 20+90, 20+10, 14+70, and 14+10, change "Leakage Paths" to "Fault Lines"; in Fig. 15(*b*), page 1425, change "10 in. on center" to "10 ft on center," in two places; on page 1431, line 9, change "3 ft to 5 ft" to "2 ft to 3 ft"; in Figs. 21(*i*) and 21(*j*), pages 1432 and 1433, interchange symbols for holes 31 and 35, and holes 32 and 30, respectively; on page 1433, line 6, change "6-in. pipe" to "3-in. pipe"; in Fig. 22(*f*), page 1434, subcaption, change "Section B-I" to "Section B"; on page 1439, line 5 from the bottom, change "No. 12 gage steel" to "No. 16 gage steel"; on page 1441, line 9, change "20-in. holes" to "20-in. casings"; in Fig. 27(*f*) change "Root" to "Rock"; and in Fig. 27(*i*) change the caption to read "(*i*) Section through hole and casing showing asbestos-cement liners and 6-in. tremie pipe."

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

# SHEAR EFFECT ON THE STRENGTH OF STRUTS

## Discussion

BY LEON BESKIN, AND MARSHALL HOLT

LEON BESKIN,[7] ASSOC. M. AM. SOC. C. E.[7a]—Extended treatment of conditions involved in Assumption 5 is superfluous, since it is accepted perforce as the conventional basis of any elementary approach to the problem of the instability of struts (see heading, "Introduction: Assumptions").

The list of assumptions is incomplete, since those leading to Eq. 2, expressing the relation of moment to curvature when the state of stress is not pure bending, and the more important assumptions leading to Eq. 9 are omitted. In that respect, it is to be noted that the values given for the constant $K$ are approximate.[8] This constant varies somewhat with the method of applying the load. It is approximately equal to 1 for I-beams, $A$ being the area of the web.

The introduction of Eq. 8 as the fundamental relationship between shear and slope is incorrect. The basic relationship for shear deflection is given by Eq. 9. The former equation is derived by integration of the latter, and is valid for straight beams on two supports only when (*a*) the end moments are zero, or are equal; and when (*b*) there are no applied moments in the span.

Moreover, in this form, Eq. 9 must be extended to include concentrated loads, by adding the equation—

$$\Delta\left(\frac{dy_v}{dx}\right) = \frac{-K\,\Delta V}{A\,G} \qquad (38)$$

—at each point of application of a concentrated load $P$, thus giving a variation of shear $\Delta V$. Eq. 38 represents a sharp angle of the neutral line of the strut. It can be derived simply by integrating Eq. 9 along an infinitely small element $dx$ on which a constant load is applied. It purports to express a variation $\Delta V$ of the shearing force, although the correct proof of its validity can only be

NOTE.—This paper by S. Sergev, Esq., was published in November, 1943, *Proceedings*.

[7] Senior Stress Analyst, Consolidated Vultee Aircraft Corp., Allentown, Pa.

[7a] Received by the Secretary December 6, 1943.

[8] "Theory of Elasticity," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1934, pp. 37, 41, 74, 104, and 150.

established from considerations of theory of elasticity.[9] By failing to make the foregoing distinction the author has developed Eq. 21, and consequently Eqs. 25 and 26, incorrectly.

For problems in which the principle of superposition does not apply, as is the case for buckling, the distribution of the deflection $y$ into two components, $y_m$ due to bending and $y_v$ due to shear, is improper. It is easily seen that, if the deflection due to bending is neglected, there is no deflection due to shear in the problem discussed. More generally, when side load exists, two conditions can be considered:

(*a*) Without longitudinal load, $y = y_m + y_v$. The deflections $y_m$ and $y_v$ are computed, assuming first that $\frac{1}{A} = 0$, and second that $\frac{1}{I} = 0$. In this case, the two components have definite meanings.

(*b*) With a longitudinal compressive load, $y > y_m + y_v$, if the two components are computed as in Condition (*a*).

The only correct relationship is

$$\frac{d^2y}{dx^2} = \frac{1}{\rho} = \frac{1}{\rho_m} + \frac{1}{\rho_v} \quad \ldots\ldots (39)$$

the two component curvatures $\frac{1}{\rho_m}$ and $\frac{1}{\rho_v}$ being defined by the right members of Eqs. 5*b* and 10*a*. Incidentally, the two quantities $y_v$ and $y_m$ that would result from the integration of Eqs. 5*b* and 10*a*, once $y$ is known, have no significance, although their sum is equal to $y$, provided they are both equal to zero at the supports.

A concentrated transverse load deforms the neutral axis of the strut. Thus, Eq. 21 $\left[ \left( \frac{dy_a}{dx_a} \right)_{x_a=a} = - \left( \frac{dy_b}{dx_b} \right)_{x_b=b} \right]$ is not correct. The shearing forces in parts $a$ and $b$ of the strut at the point of application of the load are, respectively,

$$V_a = - \frac{P\,e}{L} \quad \ldots\ldots (40a)$$

and

$$V_b = \frac{P\,e}{L} + (y_b)' \quad \ldots\ldots (40b)$$

(In Eqs. 40, $(y_a)'$ and $(y_b)'$ are the slopes on both sides of $P$.) The variation of shearing force at $P$ is

$$\Delta V = - (V_a + V_b) = - (y_b)'\,P \quad \ldots\ldots (41)$$

Then

$$- (y_b)' = (y_a)' + \frac{K\,\Delta V}{A\,G} = (y_a)' - \frac{(y_b)'\,P\,K}{A\,G} \quad \ldots\ldots (42)$$

Eq. 42 can easily be transformed into

$$(y_b)'\,m^2 = - (y_a)'\,q^2 \quad \ldots\ldots (43)$$

[9] "Theory of Elasticity," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1934, p. 103.

which should be substituted for Eq. 21. If the cross-section area changed at $P$, the calculation would be changed since $-\frac{\Delta V}{A}$ would be replaced by $\frac{V_a}{A_a} + \frac{V_b}{A_b}$. Using Eq. 43, Eq. 24*b* is changed to

$$q\,L \cot k\,q\,L = \left(\frac{q^2\,L^2}{3\,u}\right)(1-k) - \frac{1-k+\frac{q^2}{m^2}}{(1-k)^2} \quad \ldots\ldots\ldots (44)$$

It is apparent that the final equation sought by the author is modified considerably by the foregoing correction.

The constants $\gamma$ and $\psi$ are not directly related to the slope of the axis and are equal to the slope only when definite conditions are fulfilled. They define the angle between the perpendicular to the average direction of the cross section, as distorted by the effect of shear, and the neutral axis.

It is incorrect to state in a general manner that "the curvature due to bending of a built-up strut is independent of the type of lattice used" (see paragraph following Eqs. 26). For instance, the buckled shape of the members of a strut that is latticed only by batten plates is a wavy curve, and, if there are a few battens, only an average, badly defined curve will be identified with the elastic line of a solid strut. Theoretically, it is impossible to separate, definitely, the effect of shear from that of bending, and the values of $\psi$ are valid only when several conditions are satisfied. For instance, the buckling between diagonals or battens must be such as can be disregarded. A thorough investigation of the problem has been presented by Julius Ratzersdorfer.[10]

Application of Eq. 44 will change Eq. 29 as follows:

$$6\,q\,L \cot \tfrac{1}{2}\,q\,L = q^2\,L^2 - 12 - \frac{24\,q^2}{m^2} \quad \ldots\ldots\ldots\ldots (45)$$

and Eqs. 31 will be changed to:

$$6\,w \cot \tfrac{1}{2}\,w = w^2 - 12 - \frac{24\,w^2}{t^2} \quad \ldots\ldots\ldots\ldots\ldots (46)$$

in which

$$w = \frac{t}{(1-\alpha\,t^2)^{0.5}} = \frac{t}{(1-\beta\,t^2)^{0.5}} \quad \ldots\ldots\ldots\ldots\ldots (47)$$

Fig. 3 must be corrected correspondingly. The values of $\beta$ and $t$ that satisfy Eqs. 46 and 47 are easily computed when $w$ is entered as an independent variable:

$$t^2 = \frac{24\,w^2}{w^2 - 12 - 6\,w \cot \tfrac{1}{2}\,w} \quad \ldots\ldots\ldots\ldots\ldots (48a)$$

and

$$\beta = \frac{1}{t^2} - \frac{1}{w^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (48b)$$

Using the incorrect Eq. 31*b*, $t$ becomes equal to

$$t^2 = 36 + 6\,w \cot \tfrac{1}{2}\,w \quad \ldots\ldots\ldots\ldots\ldots\ldots (49)$$

[10] "Die Knickfestigkeit von Stäben und Stabwerken," by Julius Ratzersdorfer, Wien, 1936, pp. 201-243.

$\beta$ remaining the same as in Eq. 48*b*. The corrected and erroneous curves are both plotted in Fig. 5. The error can be estimated to vary between 10% and 15% of the variation of $t$ due to the effect of shear—that is, of the difference $t(\beta) - t(0)$, which is always too great in the author's calculation.

The general conclusions stated by the author are well known. The method of calculation presented is an application to a special case of the general method

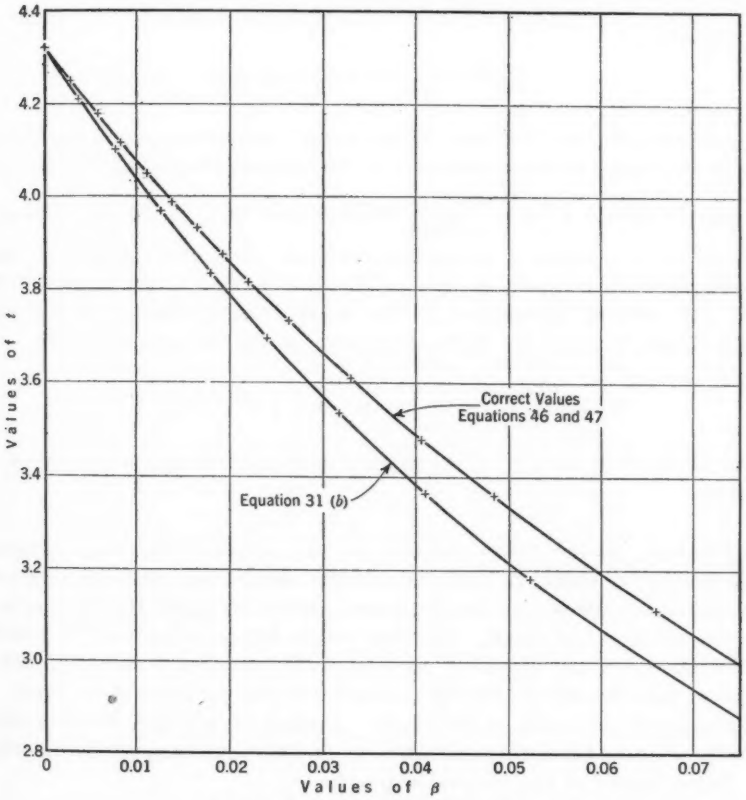


FIG. 5.—RELATIONSHIP BETWEEN $t$ AND $\beta$

originally derived by S. Timoshenko;[11] and Eq. 24*a* is the application of the general formula established by Professor Timoshenko[12] to the condition in which there is no end load:

$$\left(\frac{k_4}{k_1}\right)^2 (1 - k_1 L_1 \cot k_1 L_1) - k_1 L \cot k_1 L_1 = \left(\frac{k_2}{k_3}\right)^2 + \left[k_3 L - \frac{(k_2)^2 L_2}{k_3}\right] \cot k_3 L_2 \quad \ldots\ldots\ldots\ldots (50)$$

[11] "Theory of Elastic Stability," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1936, Paragraph 22.

[12] *Ibid.*, p. 113, Eq. 79.

In Eq. 50, when $k_1 = 0$:

$$(k_2)^2 = (k_3)^2 = \frac{P}{E\,I_b} = m^2;\ (k_4)^2 = \frac{P}{E\,I_a} = n^2;\ L_1 = a;\ \text{and}\ L_2 = b.$$

Noting that, for infinitely small values of $k_1$,

$$\frac{1 - k_1\,L_1 \cot k_1\,L_1}{(k_1)^2} = \frac{(L_1)^2}{3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

Eq. 50 becomes

$$\frac{n^2\,a^2}{3} - \frac{L}{a} = 1 + m\,a \cot m\,b \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (52)$$

As demonstrated by Professor Timoshenko,[13] the influence of the shearing force is equivalent to the replacement of the moment of inertia $I$ by a decreased moment of inertia $I\left(1 - \frac{K\,P}{A\,G}\right)$, which shows that, in Eq. 52, $m$ must be replaced by $q$, whereas $n$ remains unchanged. By substitution, Eq. 24*a* is derived directly, obviously containing the error related to the change of slope at $P$, and making unnecessary all the development presented in the paper. If the correct formula, Eq. 43, is applied to Eq. 52, the latter becomes

$$q^2\left(\frac{n^2\,a^2}{3} - \frac{L}{a}\right) = m^2\,(1 + q\,a \cot q\,b) \ldots\ldots\ldots\ldots\ldots\ldots (53)$$

which is identical to Eq. 44 when the necessary changes are made in the notation.

MARSHALL HOLT,[14] ASSOC. M. AM. SOC. C. E.[14a]—In the design of latticed struts it may be desired to have the effective slenderness ratios about the two principal axes equal so that the computed column strengths for bending in the two directions will be equal. In other words, the question may be raised as to how far apart two channels, as used in the numerical example, need be placed so that the column strengths about the axes $y$ and $z$ will be equal.

Eq. 34*a* will be useful in this study. Instead of finding a fictitious length of the member, as the author does, one can find the necessary radius of gyration $r_z$. Thus, instead of Eq. 34*c*, one can write

$$P_y = \frac{(4.32)^2\,E\,A}{\left(\frac{L}{r_y}\right)^2} = P_z = \frac{t^2\,E\,A}{\left(\frac{L}{r_z}\right)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

in which $r_y$ = the radius of gyration with respect to the $y$-axis; and $r_z$ = the required radius of gyration about the $z$-axis. It then follows that

$$\frac{r_z}{r_y} = \frac{4.32}{t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (55a)$$

[13] "Theory of Elastic Stability," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1936, p. 140.

[14] Research Engr., Aluminum Research Laboratories, New Kensington, Pa.

[14a] Received by the Secretary January 18, 1944.

or, approximately,

$$\frac{r_z}{r_y} = \frac{4.32}{4.32 - 30\beta + 160\beta^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (55b)$$

Eqs. 55 are quite like Eqs. 35. A direct solution is impossible because the value of $\beta$ depends on the size and arrangement of the lacing bars, and these have not yet been determined. A trial solution is possible which consists essentially in checking a trial value of $r_z$ by means of Eq. 55*b* or the curve of Fig. 6.

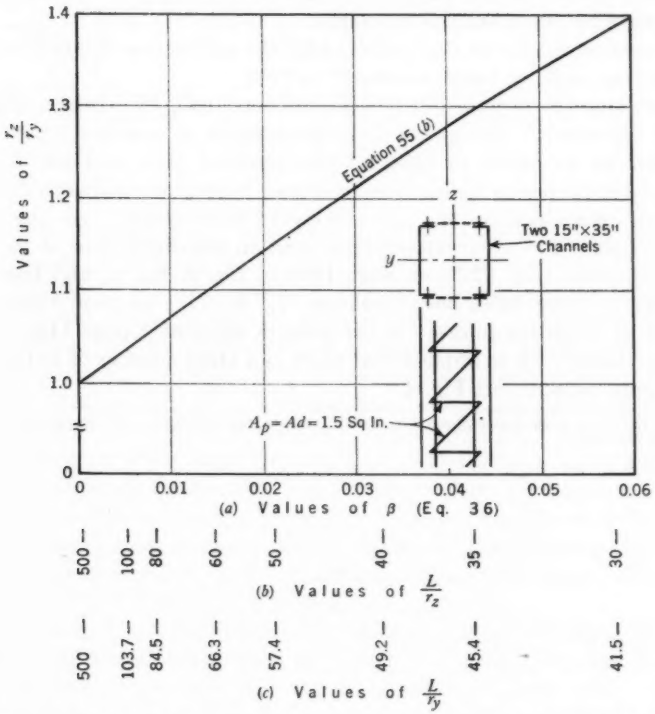


FIG. 6.—RATIO OF $r_z$ TO $r_y$ FOR EQUAL COLUMN STRENGTHS FOR BENDING ABOUT THE $z$ AND $y$ AXES

Since $\beta$ is an inverse function of slenderness ratio, Fig. 6 shows that the ratio of $r_z$ to $r_y$ should be relatively larger for small slenderness ratios than for large slenderness ratios. In other words, for a given system of lacing and a given size of lacing bar, the relation between the two radii of gyration should not be specified by a single value.

In problems of design it might be practicable to plot curves of the type shown in Fig. 6 for the system of lacing and size of bars under consideration. For this purpose, the first scale of abscissas (scale a) is in terms of $\beta$, the second (scale b) is in terms of the slenderness ratio about the $z$-axis, and the third (scale c) is in terms of the slenderness ratio about the $y$-axis. Scale c was obtained from scale b and the indicated values of the ratio $r_z/r_y$ since the

slenderness ratios are in this inverse ratio. For a certain length of column, say 320 in., the slenderness ratio about the $y$-axis is 57.4, and the required ratio of $r_z/r_y$ is 1.15. The necessary value of $r_z$ is found to be 6.41 in., and the distance back to back of the channels (14.28 in.) follows from simple computations.

In some cases it may not be desirable to have the column strength for bending about the two axes equal because of different conditions of restraint or support in the two directions. In this case a factor of proportionality of the column strengths can be introduced into Eq. 54. The procedure is the same as that for equal column strengths.

A procedure similar to that followed by the author was applied to columns loaded on the ends by Ernst Amstutz[15] in 1941.

Corrections for *Transactions:* In November, 1943, *Proceedings*, page 1447, line 5 of "Synopsis," change "is derived for struts of constant cross sections" to "is derived for struts of different but constant cross sections"; on pages 1448 and 1449, change definitions for $t$ and $\beta$ to read, respectively: "$t$ = a parameter dependent on the dimensions of the column and the material (Eq. 30)"; and "$\beta$ = a parameter dependent on the dimensions of the column and the material (Eq. 32)"; on page 1449 in Eq. 2, Eq. 3, and line 1 below Eq. 2, and on page 1450, line 3, change "$I_b$" to "$I$"; on page 1449 in line 2 below Eq. 2 delete the phrase "in the bottom segment"; page 1455, in line 1, add after "strut" "(It is implied that there is a large number of lattice bars)"; and, on page 1458, invert Fig. 4.

[15] "Die Knicklast gegliederter Stäbe," by Ernst Amstutz, *Schweizerische Bauzeitung*, Vol. 118, August 30, 1941, pp. 97–101.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

# MILITARY AIRFIELDS
# A SYMPOSIUM

## Discussion

BY JAMES B. NEWMAN, JR.

JAMES B. NEWMAN, JR.,[5] ESQ.[5a]—The development of aircraft, accelerated by war, requires modification of design and of construction of airports at every step. Only a short time ago the experienced airport engineer found no particular problem in the design of a runway pavement. His specialized knowledge was supported by the experience of hundreds of able highway engineers, and by years of accumulation of data resulting from traffic tests and scientific research. Today the problem is vastly different. Loads applied to pavements on military airfields have no precedent in either airport or highway engineering. The necessity for wide runways with comparatively flat crowns and almost negligible grades, coupled with a requirement of broad graded areas on either side and at either end of the pavement, renders drainage of airfields and of runways a much more complex problem than drainage of streets and highways, or even prewar airfields; and yet drainage, always of prime importance in design, has become increasingly critical.

In the face of the requirement of constant change of design, the engineer has been confronted with other unusual difficulties. The expansion of the Army Air Forces has exceeded the imagination of most men. This large striking power could be of no use unless large numbers of planes, manned by properly trained crews, are hurled against the enemy. The constant cry has been, therefore, for extensive work rapidly performed. There has been no time for exhaustive research, for the development of ideal methods, for the conduct of extensive surveys, and for examinations. The Corps of Engineers has been repeatedly required to complete super-airports in less than six months after the need for the developments first has been announced. There has been no opportunity for long deliberation on choice of materials. With the priorities system, the engineer has been forced to accept the materials, equipment, and man power available, and, somehow, to produce the required results.

NOTE.—This Symposium was published in January, 1944, *Proceedings*.

[5] Brigadier General; formerly Colonel, Air Corps, U. S. Army, Chf., Buildings and Grounds Section, Office of Commanding General, Army Air Force, Washington, D. C.

[5a] Received by the Secretary January 26, 1944.

That the Corps of Engineers has met the challenge of this gigantic problem is proved by the success of the construction program. The few unsatisfactory jobs have an explanation. In almost every case the result was caused by the demand for speed. Completion has been required by definite dates because units have been activated and the initiation of their training has been scheduled for those dates. Labor shortages, nonavailability of materials or equipment, rain, and freezing weather have been obstacles, but completion dates, nevertheless, have governed. In spite of these difficulties, almost without exception, the facilities have met the exacting requirements of the Army Air Forces.

It is difficult to add to the very complete outline of design and construction practices presented by Colonel Stratton. His observations are in accord not only with the practices of the Corps of Engineers, but also with Air Force policy, and are supported by Air Force experience.

Colonel Stratton has indicated the reasons for the selection of a modified California test method for the evaluation of subgrade in the design of flexible pavements. This method is subject to erratic results in tests on some of the many types of soils encountered in the United States, and further modifications of the procedure may be required in the future. The personnel of the Army Air Forces charged with the development of airport criteria and the planning of facilities is inclined to be skeptical of the California method, but has had no logical alternate to offer. More positive results might have been obtained through the use of extensive field bearing tests, but, as Colonel Stratton indicates, the time available for design and construction mitigated against this practice.

Nevertheless, plate bearing tests have a definite place in the determination of strengths of completed pavements. The training program of the Army Air Forces is far from static. Changes in the use of facilities develop suddenly and often. Installations are now numerous enough to permit some degree of flexibility in the assignment of airfields for specific purposes. Positive knowledge of actual developed strengths of pavements would insure, to a marked degree, against unnecessarily high maintenance costs and possible occasional complete failures. These tests can be made while the airfield is in operation, and, therefore, the element of time no longer takes precedence over more exact procedure.

The drainage of airfields is a subject that cannot be too strongly stressed. There is no surer way to failure than inadequate and improper drainage for wide expanses of pavement designed for heavy loads. Ponding on runways, even to a relatively minor extent, creates an operational hazard, and may result in excessive interruptions of activity. The airport engineer must master this subject, or confine his efforts to desert areas.

Engineers who have not been engaged in the design of military airports recently may have some difficulty in visualizing the loads for which pavements are designed. The lightest pavement—that for a Class IV military airfield—compares favorably with many high-grade federal highways. Several states limit truck loads to 26,000 lb gross on four wheels and 32,000 lb gross on six wheels. Yet the Class IV airfield will accommodate nothing bigger than the single-engine trainer.

The Class III airfield, intended primarily for twin-engine trainers and transports, and for most fighter types, is designed to carry two-and-one-half times the normal highway truck "wheel load."

The Class II airfield, designed for a gross load of 74,000 lb, wheel load of 37,000 lb, will accommodate the larger four-engine cargo ships fully loaded, and bombers of the Flying Fortress class. These airfields have about the same load bearing capacity as main line railroads.

Quite obviously there is no counterpart for the Class I airfield, designed to carry the heaviest planes now in production. The designed gross load of 120,000 lb and wheel load of 60,000 lb exceed any known pavement requirements; and heavier loads are to be expected in the near future.

In order to judge just how much a load 60,000 lb per wheel is, visualize the weights involved in a test made by Colonel Stratton's organization. A large size "turnpull" with large single tires was used. The rear tires had a footprint of 750 sq in. each. The front tires on the tractor weighed 6,000 lb each and had a footprint of 1,000 sq in. The "turnpull" was loaded to the top with scrap steel; then 30-ft steel rails were piled across the top until the load was exactly balanced with 50,000 lb on each of the four wheels. The tractor could pull this load at about 2½ miles per hr over the paving to be tested, with only an occasional failure of the coupling device. If this load had been amplified by four 2,000-hp engines turning propellers at high rates of speed with the resulting vibration, the wheel loads and general effect would have been more nearly equal to the loads imposed upon runways, taxiways, and aprons on Class I airfields.

Warming-up engines, locked wheel turns, and taxying at low speeds before the wings begin to lift weight and provide buoyancy are a few of the factors never before considered by paving engineers. The use of dual tires helps to solve the tiremakers' problems, and also assists in designing adequate braking surfaces; but dual tires do not lessen the load on a pavement when weights are as great as these. These examples emphasize the need for new and advanced designs, and illustrate the reason for building taxiways, warm-up blocks, and parking aprons 25% stronger than the runways constructed to the full wheel loading. No allowances are made for landing impact.

Early pilots recited a formula for locating airports which had more than an element of truth mixed with comedy: "You find a high tension line, follow it to the intersection of a railroad and highway; then look for a river or a marsh, there is your airport." The runways were to be located so that the glide paths and approach angles passed through the oldest city landmarks, the fine old trees in the central park, or the tallest smokestacks in town. When most of these early airfields were located, flying was done in small planes by "barnstormers." The locations were cheap and they grew up like "Topsy." When new requirements arose, it was always too late to remove existing facilities to proper locations.

For modern airplanes, runways must be at least 5,000 ft long at sea level. This length should be increased with increased altitude by adding 100 ft to each runway for each 200 ft of altitude measured from sea level. Some military airplanes require a basic length of 6,000 ft at sea level. Shelters have

been built for bad weather and night flying with runways up to 10,000 ft long. Different areas require different runway arrangements or patterns. The number of runways needed is controlled by wind conditions. In a few airfields, one runway is ample since the wind blows either up or down the valley. Many airports need only two runways since wind records for that area, compiled in the form of wind roses, indicate that winds blow only from four directions, each fully provided for by two runways. Most areas are amply served by three runways bearing a 60° relation to each other. A few localities definitely require four runways to cover all wind directions adequately.

Before selecting a site, the engineer should know the maximum wheel loadings and the anticipated air traffic. He should have at hand all the local wind information available. He should examine the possibility for future developments that might affect his plan. In addition, he should provide needed utilities, facilities, transportation, and all other requirements likely to influence his selection of a site. If a piece of farm land tempts him, the engineer should remember that generations of farmers have spent much time and money making the soil as unsuitable as possible for compaction and load-carrying stability. If it is level, he should remember that when Nature levels a piece of land she usually uses methods and materials which make it unsuitable for either base stability or compaction. If a parcel is level and in a cold country, the frost action may be bad, and the engineer should check all means of draining areas before selecting them. In most airfield construction the engineer has more to fear from ultimate cost of excessive fills than from large and costly cuts.

To make a proper selection he should know how many runways are needed. One will be required in each primary wind direction, or rather within 22½° of each high wind direction. Very small airplanes that would be bothered by winds blowing 10 miles per hr or less are best handled on all-over fields. Planes heavy enough to need runways can adjust themselves to fairly brisk winds crossing a runway at an angle as great as 30° off the course of such a runway. In the case of high winds, 30 miles or more, an adjustment of 22½° may be all that can be expected of either pilot or airplane, in which case a fourth runway would be required if 10% of the stiffer winds were such that three runways were not enough.

When considering runway lengths, 5,000 ft should be the minimum with 1,000-ft clear zones on each end. Some runways, plus clear zones, are 12,000 ft long. With this information, the engineer can eliminate all possible sites and start compromising, but he must not forget that the airplane should have a clear glide path not steeper than 1 on 40 from the extreme end of each clear zone and that this glide path should extend at least 2 miles beyond the extreme end of the runway clear zone. When a few possible sites have been determined, he must check the soil, the cost, and source of base-course material, in place, and, finally, the hazard removal cost and other items necessary for a decision.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

# GEOLOGY IN HIGHWAY ENGINEERING

## Discussion

BY W. W. CROSBY

W. W. CROSBY,[3] M. AM. SOC. C. E.[3a]—The plea in this paper for active cooperation of geologists in highway work interests the writer, not because he has any awareness of the lack of such cooperation, but because he personally recollects fundamental contributions by geologists to the establishment and the development of highway engineering. Long ago, the foundations were durably laid by these men who are so often forgotten when the structure is completed. Mr. Huntting's earliest reference (15)[3b] is dated 1918, and only one other (5) of the twenty eight antedates 1924.

More than two decades before, N. S. Shaler, a noted geologist and professor at Harvard University, at Cambridge, Mass., wrote "American Highways" (29) which resulted in the creation of the Massachusetts Highway Commission in 1893. It is unnecessary to detail here the outstanding leadership of that body, of which Professor Shaler was an active member.

Only a few years later (in 1898), at the suggestion of William Bullock Clark, Professor of Geology at Johns Hopkins University (Baltimore, Md.) and State Geologist, the Maryland Geological and Economic Survey was authorized to investigate "the question of road construction" in that state. The seven reports (30) of this survey (four on the "Highways of Maryland" and three on "State Highway Construction") were published in the years 1899 to 1910, inclusive, and widely distributed. The 1899 Report has been commended "as the most complete statement ever issued by any State." These reports cover in detail most, if not all, of the "problems" listed by the author and much else of at least equal importance.

Of the nine men on the "Scientific Staff" of the Maryland Geological and Economic Survey for the 1899 Report, the following were geologists: William Bullock Clark, Harry Fielding Reid, Edward B. Matthews, Charles S. Proesser, and George B. Shattuck.

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NOTE.—This paper by Marshall T. Huntting, Esq., was published in December, 1943, *Proceedings*.

[3] Cons. Engr., Coronado, Calif.

[3a] Received by the Secretary January 3, 1944.

[3b] Numerals in parentheses, thus: (15), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

Contemporaneously, the late Joseph Hyde Pratt, M. Am. Soc. C. E., as State Geologist, set the standard for modern highway work in North Carolina. Wisconsin followed the Maryland work closely with W. O. Hotchkiss, M. Am. Soc. C. E., State Geologist, supervising the groundwork. The late Logan Waller Page, M. Am. Soc. C. E., at one time Geologist for the Massachusetts Highway Commission, directed the U. S. Office of Public Roads from 1905 to 1919.

The writer has probably mentioned herein only a very few of the geologists who did so much for highway improvement, and he has scarcely hinted at the enduring character and widespread effects of what they accomplished. However, time gives a great appreciation of the fundamental, although often overlooked, importance of the work of geologists in highway engineering.

There are doubtless numerous geologists who have been, or are, contributing most valuably to the advancement of highway engineering. Indeed, a highway department cannot function without such advisers.

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(5) **"An Interesting Case of Dangerous Aggregate,"** by J. C. Pearson and G. F. Loughlin, *Proceedings*, Am. Concrete Inst., Vol. 19, 1923, pp. 142–155.

(15) **"Ice Forming in Clay Soils Will Lift Surface Weights,"** by Stephen Taber, *Engineering News-Record*, Vol. 80, No. 6, 1918, pp. 262–263.

(29) **"American Highways,"** by N. S. Shaler, The Century Co., New York, 1896.

(30) **"Reports on the Highways of Maryland,"** The Johns Hopkins Press, Baltimore, 1899–1910.

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DISCUSSIONS

# ORIGINAL AND READJUSTED REPAYMENT CONTRACTS, BOULDER CANYON PROJECT

## Discussion

BY A. R. ARLEDGE

A. R. ARLEDGE,[6] M. AM. SOC. C. E.[6a]—A record of the history of the Boulder Canyon development and the not generally known facts regarding the background of the pioneer in multiple-purpose projects are presented in this excellent paper. Although the author has mentioned the varied interests involved and the need for negotiating the contracts, further emphasis should be placed on the long negotiations for the Adjustment Act and the readjusted repayment contracts. The question may be asked: "Are not these contracts reasonably simple legal documents in which the engineering factors are either known or lend themselves readily to fairly close engineering estimates, although these factors involve long periods of time and economic factors of great magnitude?"

From the data submitted, the answer to this question does not seem so simple. The participants in the negotiations at no time considered the task an easy one; but they became more aware of the involved nature of the problems confronting them as time passed, and the pattern of the varied sectional economic and engineering difficulties assumed shape. The engineers, attorneys, and others called upon to solve these problems, of necessity, had to be men, not only informed on the subject matter, but endowed with both prophetic acumen and diplomatic dexterity of first magnitude, in order to cope with the situations as they arose.

In recognition of these difficulties, the Hearings before the Committee on Irrigation and Reclamation, United States Senate,[7] state in part, with reference to the Boulder Canyon Project Act:

> "As is hereinafter more fully pointed out, the text of the act contains many uncertainties, ambiguities, and inconsistencies."

NOTE.—This paper by Clay C. Elder was published in December, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1944, by W. L. Chadwick.

[6] Prin. Civ. Engr., Los Angeles Bureau of Power & Light, Los Angeles, Calif.

[6a] Received by the Secretary January 3, 1944.

[7] Boulder Canyon Project Adjustment Act, Hearings Before the Committee on Irrigation and Reclamation on S. 4039, United States Senate, 76 Cong., 3d Session, May 28, 1940, p. 6.

And further[8] in regard to controversies which arose:

> "The chief causes for controversy have been, first, the method of determining charges for electric energy at the dam; second, the extent of, and method of determination of, the rights of the States of Arizona and Nevada in the revenues from the project; and third, the extent of the interest of the several States of the Basin of the Colorado River, in the revenues from the dam."

The original Project Act involved the interests of the seven states of the Colorado River Basin, the United States of Mexico, and the United States of America. Although the Adjustment Act, as its name implies, is primarily an act of Congress amending the Project Act, it required the close cooperation of the seven basin states and the power contractors as represented by the Committee of 16, and the other power allottees and irrigation interests. To complete this complex picture, it is necessary to add the satellite interests which revolve around and affect each of the foregoing major groups.

In 1937, when the request for a review of the rates was filed, it was thought that the Secretary of the Interior with his broad discretionary powers could correct the difficulties in the energy rate structure as well as those encountered in administering the Project Act, but competent legal authority ruled otherwise. This ruling started, in earnest, the long and tedious negotiations which culminated in the approval of the Adjustment Act on July 19, 1940. This Act carried a provision, in substance as follows: It shall cease to be operative and shall be of no further force or effect, if contracts (the readjusted contracts of the City of Los Angeles, the Metropolitan Water District of Southern California, and the Southern California Edison Company) in accordance with the requirements of Section 10 of the Act shall not have been entered into prior to June 1, 1941. The principal function of the Adjustment Act seemed to be to provide requirements, rules, regulations, and a time limit under which the readjusted repayment contracts could be perfected. The time limit was beaten by a day.

In spite of the seemingly insurmountable difficulties and the duration of the negotiations for both the Adjustment Act, setting up requirements and limitations for the new contracts, and the new contracts, there were leveling elements which provided the incentive for a mutual agreement. First, the contractors for energy at Boulder Dam were paying a rate about 50% greater than actually was required under the conditions as stated in the Project Act. Second, there was grave doubt that the states of Arizona and Nevada would receive payments in lieu of taxes commensurate with the intent of the Project Act, or that the seven states of the Colorado River Basin would receive any payments that might be used as a development fund, because of uncertainties and ambiguities of the Act. Third, as the author has stated, there was some uncertainty, under the original Act, as to whether the federal government would be fully reimbursed in fifty years because of the requirement to readjust rates periodically in accordance with rates prevailing at competitive centers.

The leveling elements, which provided each main group with trading points, under certain conditions may have given the contractors of energy some slight

[8] Boulder Canyon Project Adjustment Act, Hearings Before the Committee on Irrigation and Reclamation on S. 4039, United States Senate, 76 Cong., 3d Session, May 28, 1940, p. 7.

advantage. The "Contract for Lease of Power Privilege," under date of April 26, 1930, in Article 16, has the following provision:

> "At the end of fifteen (15) years from the date of execution of this contract and every ten (10) years thereafter, the above rates of payment for firm and secondary energy shall be readjusted upon demand of any party hereto, either upward or downward as to price, as the Secretary may find to be justified by competitive conditions at distributing points or competitive centers."

Under this provision, a readjustment of rates would have taken place automatically in 1945. Unless the prices of fuel oil and gas had increased materially, the rates for firm and secondary energy, probably, would have been too low to guarantee the repayment of the federal government's advances in fifty years, and also to make payments to the states of Arizona and Nevada, and to the Colorado River Development Fund to be used in the interest of developing a comprehensive plan for the Colorado River Basin as a whole. If the contractors had chosen this alternate, they would have been penalized by the payment of many millions of dollars in excess revenues before 1945. It seems that the best long-term plan has been accepted, measured in terms of benefits to the entire basin.

Engineers may not be in agreement on the merits of one project subsidizing other projects, but, perhaps, all will agree that, to develop a contract extending fifty years into the future, under the present conditions of social and economic flux, it required men of vision and resourcefulness in each major group, working earnestly and diligently for a comprehensive plan of development of the Colorado River Basin.

Correction for *Transactions:* In January, 1944, *Proceedings*, page 124, change the tabulated information for generating units N-1 to N-4 to read as follows:

Nevada Side—

| | |
|---|---|
| N-1, N-2, N-3, N-4 | City of Los Angeles, Burbank, Glendale, Pasadena, Boulder City, California Pacific Utility Company, and Citizens Utility Company (contractors for the Metropolitan Water District), and the State of Nevada for the Southern Nevada Power Company and the Lincoln County Power District No. 1 |

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DISCUSSIONS

# STRENGTH OF SLENDER BEAMS

## Discussion

BY H. D. HUSSEY

H. D. HUSSEY,[7] M. AM. SOC. C. E.[7a]—It is interesting to compare the beam formulas in current use with the secant formula, Eq. 37. Beam formulas from current specifications are as follows: American Railway Engineering Association and American Association of State Highway Officials specifications for bridges,

$$s_b = 18{,}000 - 5\frac{L^2}{b^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (80a)$$

and American Institute of Steel Construction specifications for buildings (maximum = 20,000),

$$s_b = \frac{22{,}500}{1 + \dfrac{L^2}{1{,}800\, b^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (80b)$$

in which $s_b$ is the maximum allowable compressive unit stress in bending.

If the bridge formula, Eq. 80*a*, is multiplied by $\frac{28.8}{18}$, and the building formula, Eq. 80*b*, by $\frac{28.8}{20}$, ultimate stresses may be obtained which will be consistent with those presented by the author in Fig. 4. Fig. 7 shows the ultimate stresses thus derived from the bridge and building formulas when plotted in Fig. 4.

Thus the equation for curve E, Fig. 7, is:

$$\bar{s} = \frac{28.8}{18}\left(18{,}000 - 5\frac{L^2}{b^2}\right) = 28{,}800 - 8\frac{L^2}{b^2} \quad \ldots\ldots\ldots\ldots (81a)$$

NOTE.—This paper by George Winter, Esq., was published in June, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by Messrs. T. McLean Jasper, and C. M. Goodrich; and December, 1943, by J. P. Wadleck.

[7] Designing Engr., Am. Bridge Co., New York, N. Y.

[7a] Received by the Secretary January 8, 1944.

and, for curve F, Fig. 7 (maximum, 20,000):

$$\bar{s} = \frac{28.8}{20}\left[\frac{22{,}500}{1 + \dfrac{L^2}{1{,}800\,b^2}}\right] = \frac{32{,}400}{1 + \dfrac{L^2}{1{,}800\,b^2}} \quad \ldots\ldots\ldots\ldots (81b)$$

For $L/b = 30$, Eqs. 81*a* and 81*b* give identical results.

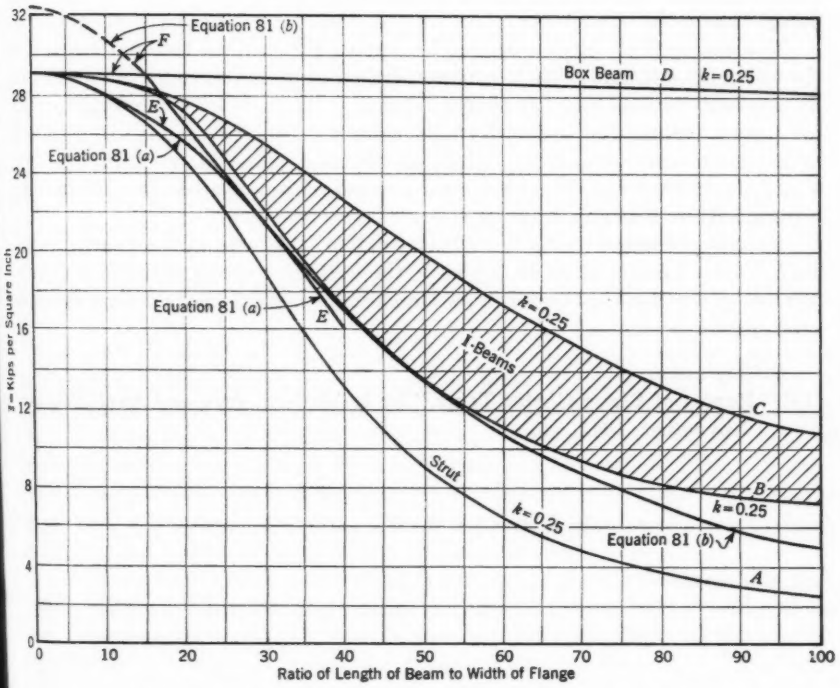


FIG. 7.—ULTIMATE STRESS $\bar{s}$ FOR A YIELD POINT OF 36,000 LB PER SQ IN. AND FOR $k = 0.25$ (COMPARE FIG. 4)

Fig. 7 indicates that the present design formulas are conservative. The close correspondence between the curves representing the present design formulas and the author's results suggests that the tentative value of $k = 0.25$ may be a satisfactory assumption until the correct value can be established by tests.

The author has proved the great torsional strength of box beams, theoretically, and has confirmed the results of tests reported by I. Madsen (14)[7b] in 1941. One of Mr. Madsen's conclusions was: "For welded (box) girders with $L/b$ ratios up to 110, and riveted (box) girders with $L/b$ ratios up to 80, it is not necessary to reduce the allowable compressive design stress." This statement seemed rather startling until Professor Winter gave it a

[7b] Numerals in parentheses, thus: (14), refer to corresponding items in the Bibliography (Appendix I of the paper), and at the end of discussion in this issue.

theoretical background, calculated the ultimate stresses for a box beam for varying values of $L/b$, and plotted them in Fig. 4. This beam, which has a maximum strength of 28,800 lb per sq in. when $L/b = 0$, has a strength of more than 28,000 lb per sq in. when $L/b = 100$. Current specifications make no distinction between the flanges of a box girder and an I-beam, as regards allowable flange stress. The same formula is used for both sections. Present evidence seems to justify a review of current practice in respect to allowable flange stresses in box girders.

The author is to be commended for having presented a rational solution of the strength of beams. He has demonstrated that this strength is represented by a secant formula and has thereby developed a basis for a working formula for beams. Since the secant formula cannot be solved readily, it would be necessary to use an approximation, as has been done for column formulas. Fig. 7 shows that Eqs. 81 give a good approximation of a secant curve for the ultimate stress $\bar{s}$ at any value of $L/b$.

Results presented in Fig. 7 indicate that (*a*) it is safe to use beams having an $L/b$ ratio greater than 40, which is the currently specified limit; and (*b*) a formula of the Rankin-Gordon type would define a satisfactory design stress for such slender beams.

*Bibliography.—*

(14) **"Report of Crane Girder Tests,"** by I. Madsen, *Iron and Steel Engineer*, November, 1941.

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DISCUSSIONS

# DEWATERING, INCINERATION, AND USE OF SEWAGE SLUDGE

## A SYMPOSIUM

### Discussion

BY LANGDON PEARSE

LANGDON PEARSE,[12] M. AM. SOC. C. E.[12a]—The range of procedure presented in this Symposium illustrates the rapid advance in the practice of handling sewage sludge since the early 1920's, and particularly in the technique of conditioning, dewatering, and heat-drying sludge. The discovery of the value of ferric chloride as a conditioner by Mr. Palmer and the extended study by Mr. Dundas and his associates in heat-drying and incineration are among the most important advances in the art. The first stage of the development related to design; the second, to operation and maintenance.

Mr. Velzy has described operating conditions at the Buffalo installation, in which primary sedimentation produces a sludge that is treated by partial digestion, vacuum filtration, and incineration. Mr. Fuhrman discusses the plant of the District of Columbia, where the primary sludge is digested and elutriated before vacuum filtration, the cake being dumped. Mr. Schroepfer analyzes the Minneapolis-St. Paul plant, in which sludge from primary settling is concentrated, dewatered, and incinerated. The maintenance problems in each are somewhat different: Buffalo uses the flash dryer and Minneapolis-St. Paul, the multiple hearth; Mineapolis-St. Paul does not employ digestion, Buffalo uses partial digestion, and the District of Columbia has complete digestion with elutriation of sludge. The problem of The Sanitary District of Chicago has been the design and operation of apparatus to dewater, heat dry, and incinerate excess activated sludge, as described by Messrs. Dundas and McLaughlin.

If the end product is fertilizer, certain additional costs are incurred from the need of fuel to replace the heat otherwise supplied by the sludge itself when

NOTE.—This Symposium was published in January, 1943, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1943, by A. L. Genter, M. Am. Soc. C. E.; June, 1943, by Willem Rudolfs, M. Am. Soc. C. E.; and September, 1943, by Messrs. Lloyd M. Johnson, and Isador W. Mendelsohn.

[12] San. Engr., The San. Dist. of Chicago, Chicago, Ill.

[12a] Received by the Secretary November 29, 1943.

burned. These costs have been estimated at $2.50 to $3.00 per ton of dried sludge. In the operation of the equipment, a drier filter cake can be secured if incineration follows. This affects the output of the equipment, measured in dry weight. Furthermore, the rates of filtration on a vacuum filter are considerably lower, measured in pounds of dry solid in cake per hour, for activated sludge than for fresh or digested sludge, and the moisture content of the cake is higher.

For filter cloths Buffalo and Minneapolis-St. Paul use canton flannel. The District of Columbia and Chicago use woolen cloth. At Chicago wool was chosen, despite the higher first cost, because with a life of three months the total cost of the woolen cloth in place was less per ton of product. Because of war conditions, various materials have been tried for winding wire. However, a method of crimping the cloth in place with grooves and wooden splines is proving satisfactory, without any use of wire. The pipe drainage system of the filters is being replaced with plastic or glass pipe. Wooden backing has proved superior to wire mesh.

Heat-dried activated sludge at Chicago is sold only in bulk in carload lots, based on analysis, under a 5-yr contract with an exclusive buyer. The material is distributed chiefly in the southeastern United States. The output has varied. Prior to World War II, the equipment was worked at a higher rate than proved feasible under war conditions. The sales are shown in Table 20.

TABLE 20.—SALES OF HEAT-DRIED ACTIVATED SLUDGE AT CHICAGO

| Year | Tons | Percentage, Dry Weight | |
|---|---|---|---|
| | | Ammonia | $P_2O_5$ |
| 1939 | 11,303 | 6.00 | .... |
| 1940 | 58,464 | 6.40 | 2.25 |
| 1941 | 56,597 | 6.24 | 2.27 |
| 1942 | 51,729 | 5.97 | 2.45 |

The ammonia content at the Southwest Works at times is as high as 8.1%, but at no time has it fallen below 4.37%. The Calumet sludge is consistently lower in ammonia than the Southwest sludge.

At present there is considerable demand for organic material for mixing purposes. Milwaukee and Chicago are currently selling all they can produce. Milwaukee markets about half its output in bags through jobbers and retailers. This is practicable because of storage facilities of about 25,000-ton capacity. At Chicago, shipment is made as manufactured.

The status of the conversion of sewage sludge to fertilizer is apparently a matter of economics. At present, with the rigid control over chemical nitrogen, organic nitrogen is in demand. From 1910 to 1920, the trend of the organic nitrogen market was upward, reaching a peak in 1920. It then turned downward, largely because of the competition of chemical fertilizers. The trend was reversed about 1932 and rose until 1937, then dropped slightly, and finally stabilized at a somewhat higher level for the war period under ceiling control. Undoubtedly organic matter or humus is required in many soils. Heat-dried activated sludge is the most valuable of the sludges. However, other sludges properly prepared offer a useful source of humus and nitrogen of lower grade. In general, the distribution of sludge in agriculture is limited largely by economic reasons. Air-dried digested sludge is suitable for local use, as a sub-

stitute for manure, whereas heat-dried activated sludge may be shipped as far as 1,000 miles.

Because of the uncertainty of prices for organic nitrogen in postwar times, the planner of large activated sludge works should certainly provide another outlet than fertilizer. The plants of the Sanitary District are designed either to burn the heat-dried sludge or to sell it as fertilizer. Because war conditions prevent any present installation of additional equipment, the Sanitary District has just completed a 16-in. sludge pipe line, $4\frac{2}{3}$ miles long, from the Southwest Works feeding a system of lagoons, covering 78 acres, divided into 16 units. The working depth averages 16 ft. This will handle excess activated sludge seeded with Imhoff digested sludge.

The use of digestion tanks as a buffer between the settling tanks and the vacuum filter may be necessary in many projects to average the solid load and reduce the equipment for vacuum filters and incinerators. However, situations may arise in which digestion may prove undesirable, particularly in handling industrial wastes from the brass industries, containing copper, which tend to slow up or stop the digestion of sludge. In such cases the designer should consider taking the raw sludge direct from the settling basin to the vacuum filter, with suitable conditioning followed by incineration.

Although the Sanitary District has given considerable thought to the disposal of fly ash, as yet nothing has developed which replaces the pumping of a wet slurry of fly ash to a neighboring lagoon. At the Southwest plant the fly ash is practically all derived from the burning of coal in making steam as well as for drying.

From the standpoint of operation and maintenance, there are still opportunities for improvement in the details of design. Both capacity and life of equipment should be increased, with a minimum of expensive or hard-to-get material. In the conditioning of sludge, elutriation has come forward, particularly for handling digested sludge. In the field of conditioning agents, ferric chloride still is preeminent. Thus, the chemical industry has materially aided sewage treatment development by substantial reductions in price since 1925.

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DISCUSSIONS

# SLUDGE DRYING DEVELOPMENTS AT CHICAGO, ILL.

## Discussion

BY LANGDON PEARSE

LANGDON PEARSE,[4] M. AM. SOC. C. E.[4a]—Sludge drying development at Chicago has been presented briefly by Mr. Johnson. The West Side (Imhoff tank) plant provides a sludge digestion volume of 10,532,000 cu ft. It was expected to accommodate both the sedimentation sludge from the West Side and the excess activated sludge from the North Side. This proved inadequate, because the excess activated sludge would not concentrate within reasonable limits.

The sludge-drying beds were provided for the West Side only, with the expectation of immediate enlargement. The sludge stripping machines are

TABLE 4.—SLUDGE

| Line No. | Sewage works | (a) PLANT LOADING | | | | |
|---|---|---|---|---|---|---|
| | | Million Gallons Daily | | Population | | |
| | | 1941 | 1942 | 1940 census | 1941[a] | 1942[a] |
| 1 | Calumet[e] | 66.4 | 82.4 | 327,092 | 313,800 | 377,500 |
| 2 | North Side[c] | 206.8 | 195.8 | 1,019,967 | 1,184,000 | 1,122,000 |
| 3 | Southwest[c] | 338.0 | 331.0 | 1,127,682 | 2,770,000 | 3,014,000 |
| 4 | West Side[d] | 453.0 | 423.0 | 1,487,773 | 2,244,000 | 2,180,000 |
| 5 | Total | 1,064.2[e] | 1,032.2[e] | 3,962,514[f] | 6,512,300[g] | 6,780,800[g] |
| 6 | Over-all[e] | .... | .... | .... | .... | .... |

[a] Total equivalent population. [b] Including grit. [c] Activated sludge. [d] Imhoff tanks. [e] For four major plants

unique.[5] The Sanitary District owns and operates four large and two small locomotives and forty-one side-opening dump cars for moving the air-dried sludge to a dump some 3 miles away. By moving the sludge in a wetter condition than heretofore, the output of the drying beds has been increased materially.

NOTE.—This paper by Lloyd M. Johnson, Esq., was published in September, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by C. W. Gordon, and October, 1943, by Paul Hansen.

[4] San. Engr., The San. Dist. of Chicago, Chicago, Ill.

[4a] Received by the Secretary November 29, 1943.

[5] *Engineering News-Record*, January 21, 1932, pp. 82–85.

The Southwest works was left unfinished in 1939 because of lack of money. Two years later, when bonding power became available, construction was stopped because of the war.

The most recent data on the plant loading and over-all performance of each plant are given in Tables 4(*a*) and 4(*b*). The suspended solids record for the four plants was as shown in Table 4(*c*). At the West Side works, the record of dried sludge and scum removed from the drying beds was as given in Table 5(*a*).

In the operation of the Southwest works, certain changes have occurred. In the vacuum filters, plastic pipe is replacing the metal pipe in the pipe drainage system. The wire mesh underdrainage supporting the filter cloth is now replaced by a wooden drainage grid. The winding wire is being discarded for a system of crimping the cloth in grooves with locking splines.

Lately, the filter cake has been ranging from 83% to 86% moisture. This is fairly representative for past seasons at Milwaukee, Wis. Fly ash is disposed of in lagoons near the plant. It lacks commercial value.

Recently the grease collected from the preliminary tanks at the Southwest works was sold for 0.55¢ per lb, f.o.b. trucks at the works. The output is estimated from 5,000 to 20,000 lb of scum per day, with an average of 14,000 lb. This contains approximately 50% grease, of which 15% is unsaponifiable. The heat-dried activated sludge has been sold in bulk under a five-year contract to an exclusive purchaser, on an analysis basis. The average results are given in Table 5(*b*).

When sludge is sold as fertilizer, there is an additional expense to the Sanitary District for fuel and incidental charges, which reduces the gross re-

DRYING AT CHICAGO, ILL.

| (*b*) PERCENTAGE REMOVAL | | | | (*c*) TONS OF SOLIDS: | | | | Line No. |
|---|---|---|---|---|---|---|---|---|
| Suspended Solids | | Five-Day B.O.D. | | Arriving | | Removed[b] | | |
| 1941 | 1942 | 1941 | 1942 | 1941 | 1942 | 1941 | 1942 | |
| 90.8 | 90.7 | 90.5 | 90.2 | 13,051 | 16,075 | 11,836 | 14,541 | 1 |
| 93.8 | 92.8 | 94.9 | 94.5 | 45,782 | 41,279 | 40,529 | 38,156 | 2 |
| 64.9 | 62.4 | 56.1 | 56.5 | 105,038 | 103,066 | 67,405 | 48,204 | 3 |
| 54.5 | 51.2 | 44.3 | 44.7 | 102,182 | 107,014 | 38,249 | 44,485 | 4 |
| .... | .... | .... | .... | 266,055 | 267,434 | 158,019 | 145,388 | 5 |
| 67.5 | 65.7 | 60.6 | 61.5 | .... | .... | .... | .... | 6 |

[a] Entire Sanitary District. [b] Including miscellaneous plants and corn products waste.

ceipts. At the Calumet works in 1941 this cost slightly exceeded the receipts. In 1943 the receipts will be somewhat greater than in 1942.

To take care of the large sludge load at the West-Southwest works, the Sanitary District has completed a 16-in. sludge pipe line, 4⅔ miles long, to discharge excess activated sludge for the duration of the war into 78 acres of lagoons, 16 ft deep, divided into 16 units. This will provide about 45,700,000 cu ft of storage. Sludge will be seeded with digested sludge from the West Side Imhoff tanks. The heat-drying plant will be operated to capacity. Any excess sludge will go to the lagoons. With the new blower just completed and

with rehabilitated air-diffuser plates, the treatment of 300 mgd of sewage by activated sludge should be reached next spring at the Southwest works.

Among the problems of the sewage works designer and operator is the tuning up of the works. The varied works of the Sanitary District have shown a marked difference. The West Side works went into service with but little difficulty. The North Side works also began operating smoothly, and con-

TABLE 5.—ANALYSIS OF DRIED SLUDGE

| Year | (a) West Side Works | | | | | (b) Southwest Works | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Cubic Yards | | Average Analysis (%) | | | Tons delivered | Receipts, in dollars | Average Analysis | |
| | Scum | Sludge | Moisture | Volatile | Nitrogen | | | Ammonia | $P_2O_5$ |
| 1936 | 577 | 36,158 | 42.0 | 41.8 | 1.96 | .... | .... | .... | .... |
| 1937 | .... | 72,379 | 52.9 | 42.1 | 1.79 | .... | .... | .... | .... |
| 1938 | .... | 83,156 | 54.0 | 43.8 | 1.82 | .... | .... | .... | .... |
| 1939 | 2,064 | 61,253 | 49.5 | 42.5 | 1.91 | 11,303 | 71,102 | 6.00 | .... |
| 1940 | .... | 129,064 | 65.7 | 44.0 | 2.12 | 58,464 | 346,307 | 6.40 | 2.25 |
| 1941 | 9,134 | 149,867 | 65.1 | 47.1 | 2.03 | 56,597 | 282,428 | 6.24 | 2.27 |
| 1942 | 8,666 | 164,973 | 70.3 | 44.3 | 2.22 | 51,729 | 391,386 | 5.97 | 2.45 |

tinued with the least operating problems of any of the activated sludge works of the Sanitary District. This is due in part to the transfer of the sludge disposal to the Southwest-West Side works and in part to the character of the sewage.

The Calumet works was used as a pilot plant for the Southwest works in so far as dewatering, heat drying, and incineration were concerned. Various changes were made over a period of some three years before a complete working plant resulted. Industrial wastes presented other difficulties. The removal of waste-iron pickling liquor from the system greatly reduced the clogging of air diffusers.

The Southwest works began operation in May, 1939, as a partly completed plant. It remains such today. It is unable to give activated sludge treatment to the entire flow because of lack of air compressors. Further, it lacks adequate sludge-handling equipment. The diffusers have now become clogged to a degree requiring replacement in 1944. The air-cleaning devices need rehabilitation. Suitable grit chambers are required, as none were originally provided. The designers were handicapped by lack of funds. When funds became available, war conditions stopped construction. The operating force has done its best to cope with the difficulties of operating under war conditions.

Those interested can follow the various problems in the Annual Budget Report of the Chief Engineer of The Sanitary District of Chicago for 1940–1943, inclusive. The period of tuning up and rehabilitation of the Southwest works apparently will continue until several years after the end of the war. Meanwhile the plans for completion of the West-Southwest works are in process.

This discussion is intended to add certain phases of the sludge-handling developments, which Mr. Johnson has omitted, to complete the picture of the sewage treatment problem at Chicago and the relation of the sludge handling to it. The recent developments are noted in the sale of scum, in construction of lagoons and pipe line, and in disposal of fly ash, as well as in the difficulties in tuning up and operating an incomplete Southwest works under war conditions.

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DISCUSSIONS

# STORAGE AND THE UNIT HYDROGRAPH

## Discussion

BY JAMES S. SWEET, AND OTTO H. MEYER

JAMES S. SWEET,[10] ASSOC. M. AM. SOC. C. E.[10a]—The relationship between flow and storage in open channels has been demonstrated in this paper. The analysis and the suggested procedures of the flow routing are a definite contribution to flood control design estimates. However, in forecasting work, where time of preparation and issuance of a forecast must necessarily be reduced to a minimum, the procedure must be simplified to effect a quick answer, without sacrificing the limits of relative accuracy. Mr. Clark's method involves time-consuming, laborious computations, and therefore is not adaptable for this type of work. On the James River where stage relationships between the gages along the entire length of the river are not always well defined, due to the nonuniformity of the distribution of the storms over the basin, it is best to use the method of the summation and routing of the discharges. Discharges in the upper reaches of the main stem and the tributaries should be ascertained either through the direct reports from the stations or through the simple rainfall-stage relations and should be routed to a downstream point on the main stem. Adjustments should be made to synchronize the arrival of the discharges at the desired point simultaneously. The discharges from the intervening ungaged areas should be adjusted, taking into consideration the ratio of index areas and rainfall over them and the area coefficient that reflects runoff characteristics and channel storage of the area. Determined empirically, these coefficients for various parts of the basin range from 0.6 to 0.7. Thus, discharge of the James River at Buchanan may be expressed as follows:

$$Q_B = Q_L + Q_P + Q_F + Q_U \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which $Q_B$ = discharge of the James River at Buchanan; $Q_L$ = discharge of the James River at Lick Run, 9 hours previously; $Q_P$ = discharge of Craig Creek at Parr, 7 hours previously; $Q_F$ = discharge of Catawba Creek at

NOTE.—This paper by C. O. Clark, Jun. Am. Soc. C. E., was published in November, 1943, *Proceedings*.

[10] Regional Hydrologic Engr., U. S. Weather Bureau, Flushing, N. Y.

[10a] Received by the Secretary December 20, 1943.

Fincastle, 5 hours previously; and $Q_U$ = discharge from the ungaged area (280 sq miles) above Buchanan.

For uniform storms, using the discharge at Lick Run as an index for the ungaged area discharge, Eq. 15 is further simplified:

$$Q_B = 1.123\,Q_L + Q_P + Q_F \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

Discharges, including peak discharges, can be determined in a similar manner at all desired points along the main stem and converted into stages. With the equations prepared in a simple form such as Eqs. 15 and 16, actual computations take very little time after the headwater discharges become available to the forecaster.

It is regretted that Mr. Clark is introducing a new concept of time of concentration, thus adding another to the multitude of existing definitions. Not in the hope of relieving the confusion, but rather to add another choice for the future selection of an appropriate definition, the writer suggests that consideration be given to the definition of concentration time as time elapsed between the beginning of the effective rainfall (rainfall after the initial loss is satisfied) and the time of the peak rise of the stream.

OTTO H. MEYER,[11] ASSOC. M. AM. SOC. C. E.[11a]—A new tool for synthesizing runoff from rainfall is presented in this paper. It is a good and useful tool, as is shown by the excellent agreement of reconstituted hydrographs with natural hydrographs. Although this method will not replace other accepted techniques of synthesis entirely, it deserves to be used in a large proportion of cases because of both its simplicity and accuracy.

The author does well to emphasize relative accuracy. Estimates have often been condemned for lack of accuracy when they were in fact the best available.

Some conclusions, in the paper, such as increase in outflow following decrease in inflow, mass rotation of water-surface profiles, and negative outflows, are apparently deduced from empirical and approximate formulas. These conclusions should be examined critically. Also, the statement that $K$ is constant (and the derivation of formulas based on that condition) involves rationalizing from an empirical assumption. The value of $K$ can often be assumed constant, with good "relative accuracy" and in many cases with entirely satisfactory results; yet this value usually varies somewhat with $S$ and $Q$; in fact, it is often so variable as to be unusable. The writer has developed numerous storage-outflow curves for streams in the Sacramento and San Joaquin valleys of California, and for good routing results these curved considerably; in several cases, there were sharp reversals of curvature. The principal effect of using a constant $K$, instead of a variable $K$, will probably be the shifting of high peaks in time relative to lower flows.

Mr. Clark mentions storage as "balanced" or "unbalanced." These two types of storage appear to refer to "reach" and "reservoir" storage, respectively.

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[11a] Received by the Secretary January 17, 1944.

The division of runoff into surface and ground-water flow, by the criterion that the runoff in direct proportion to storage is surface runoff, is arbitrary. This theory is permissible only for convenience or when no other means of division is practicable. An important part of the runoff other than surface runoff never reaches the true ground-water table, but flows quite near the surface, and responds promptly to rainfall. The writer has found an apparent correlation between this volume in subsurface storage and the infiltration from rainfall; that is, the subsurface storage determines not only the outflow but also the surface runoff factor. Thus, a part of the rainfall may be routed "through the ground"; but as it travels underground a short distance only (at most several hundred yards) the outflow should be added to the "net" pluviograph and synthesized into the hydrograph along with the direct surface runoff. Of course, the inflow into subsurface storage is not total rainfall less direct surface runoff, as evaporation must also be considered; likewise, some of the subsurface storage is lost to the true ground water, and some is lost in transpiration.

The author constructs a time-area concentration curve, routes it through storage, and averages the outflow over a unit time to form a unitgraph. If the time-area concentration curve were averaged over a unit period, routing of the resulting graph might produce the same unitgraph. To extend this idea further, a summation of the products of a time-area concentration curve with successive parts of a net pluviograph could be routed through the storage with about the same labor as by the author's procedure. Although the suggested method would have the advantage of permitting use of a storage-outflow relation involving a variable $K$, it would involve more routing. However, there are less laborious procedures than the Muskingum method, such as "lag routing,"[12] or the use of a flood routing machine.

The application of Mr. Clark's technique is not necessarily limited to the Muskingum method of routing, but can be used with any of about a half-dozen methods in current use.

Correction for *Transactions:* In November, 1943, *Proceedings*, page 1341, line following Eq. 6*b*, change "Eq. 5*b*" to "Eq. 6*b*."

[12] "Simplified Flood Routing," by Otto H. Meyer, *Civil Engineering*, May, 1941, p. 306.

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DISCUSSIONS

# EFFECT OF TURBULENCE ON SEDIMENTATION

## Discussion

BY R. SCHUHMANN, JR.

R. SCHUHMANN, JR.,[12] ESQ.[12a]—The treatment of the effect of turbulence on sedimentation, presented in this paper, will be of widespread general value because, both experimentally and theoretically, the simplest possible turbulent-sedimentation system was considered. That is, the particles were in a dispersed state and of one size and one settling velocity; the suspensions were dilute; agitation was uniform with respect to depth; and there were no horizontal currents. Because of the complicating effects of variable particle size, flocculation, concentrated suspensions, variable turbulence, etc., only empirical data can be obtained in practicable sedimentation systems.

The sedimentation systems with which the writer is familiar occur in separation operations for dressing minerals. These operations pertain not only to the separation of suspended matter from fluids but also to the grading of particles according to size, and to the separation of particles of one mineral from those of another when the minerals differ in specific gravity. Turbulence, induced both by flow and by mechanical agitation, undoubtedly plays a large rôle in some of these operations, and Ensign Dobbins' methods seem to afford a good starting point for fundamental study.

The writer is particularly interested in the observations on pickup, since various devices for separating minerals require maintaining conditions of selective pickup at the bottom of a stream. Gold, cassiterite, and other heavy minerals are separated from quartz and other light gangue minerals in this way. In some respects the "rheolaveur-launder" process for separating coal and slate is also of the same nature.

NOTE.—This paper by William E. Dobbins, Jun. Am. Soc. C. E., was published in February, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1943, by A. M. Gaudin; May, 1943, by John S. McNown; June, 1943, by Messrs. H. A. Einstein, and Thomas R. Camp; October, 1943, by Messrs. Garbis H. Keulegan, C. W. Thornthwaite, and A. A. Kalinske; and January, 1944, by E. R. Van Driest.

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[12a] Received by the Secretary January 18, 1944.